Cross-sectional uncertainty and the business cycle: evidence from 40 years of options data

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Abstract

This paper presents a novel and unique measure of cross-sectional uncertainty constructed from stock options on individual firms. Cross-sectional uncertainty varied little between 1980 and 1995, and subsequently had three distinct peaks – during the tech boom, the financial crisis, and the coronavirus epidemic. Cross-sectional uncertainty has had a mixed relationship with overall economic activity, and aggregate uncertainty is much more powerful for forecasting aggregate growth. The data and moments can be used to calibrate and test structural models of the effects of uncertainty shocks. In international data, we find similar dynamics and a strong common factor in cross-sectional uncertainty.

1 Introduction

This paper reports a novel option-implied measure of cross-sectional uncertainty. Whereas the VIX, the most widely used option-implied uncertainty index, measures uncertainty about the state of the aggregate stock market (and, potentially, economy), we construct an index that tracks uncertainty about the cross-sectional distribution of firm outcomes. In many recent models and empirical analyses, it is precisely the cross-sectional component that is the critical driving force.[1]

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[1]Bewley (1986), Bloom (2009), Kaplan and Violante (2014), Christiano, Motto, and Rostagno (2014), Schaal (2017), Ilut, Kehrig, and Schneider (2018), and Arellano, Bai, and Kehoe (2019).

More formally, one might decompose the shock to a firm, $\eta_{i,t}$, into an aggregate component, μ_t, and an orthogonal component, $\varepsilon_{i,t}$ (which may be correlated across subsets of firms):

$$\eta_{i,t} = \mu_t + \varepsilon_{i,t} \tag{1}$$

The *total uncertainty* a firm faces is measured by the conditional (time-t) variance of $\eta_{i,t+1}$. The VIX and other measures of *aggregate uncertainty* capture the conditional variance of μ_{t+1}. Finally, *cross-sectional uncertainty*, on which this paper focuses, is measured by the conditional variance of $\varepsilon_{i,t+1}$: it is the variance of the shocks faced by firms that are orthogonal to aggregate shocks. We measure cross-sectional uncertainty similarly to the VIX, using option-implied volatilities.

Our cross-sectional uncertainty measure is simple to construct: it is just average firm-level option-implied conditional variance minus market implied conditional variance ($\mathrm{var}_t(\eta_{i,t+1}) - \mathrm{var}_t(\mu_{t+1})$). Under general conditions, that gap measures the average conditional variance of the residual from a regression of each stock's return on the market return. Because it is constructed from market prices, our measure is forward-looking, and is available continuously, in real time, making it particularly useful for policymakers, in addition to being much more suitable for estimation than, for example, the annual data on dispersion that is available for TFP. In addition, the measure is available for a long span of time (40 years), including six recessions. Past work on firm-level option-implied uncertainty has at most extended to 1996, observing only two business cycles.

In this paper, we document several empirical patterns in the relationship between our new measure of cross-sectional uncertainty and the economy. We focus on two types of patterns: the cyclical behavior of cross-sectional uncertainty and the forecasting power of cross-sectional uncertainty for future economic activity. We find that cross-sectional uncertainty has a mixed relationship with the state of the business cycle, rising during the tech boom of the late 1990's, but also during the financial crisis and coronavirus epidemic. The dark line in figure 1a plots cross-sectional uncertainty. From the start of our data in 1980, up to 1995, there was surprisingly little variation. After 1995, firm-level uncertainty moves much more (though still less than market uncertainty, in proportional terms), with three distinct increases, during the tech boom, the financial crisis, and the coronavirus epidemic. In the three episodes where uncertainty is elevated, it rapidly declines, returning to its long-run average by the trough of the recession. In a shorter sample, international data displays similar behavior and also has a very strong factor structure, implying that cross-sectional uncertainty is driven by global shocks.

Overall, the data appears to show that cross-sectional uncertainty is sometimes high in bad times, and sometimes high in good times. Two different classes of models exist that predict one or the other behavior for cross-sectional uncertainty, but not both. The financial crisis, with low activity and high uncertainty, is consistent with the models that emphasize countercyclical uncertainty, whether it is an endogenous response or an exogenous shock. Interestingly, though, if output tracked cross-sectional uncertainty over time, it would have recovered from the financial crisis by 2010 (when unemployment was still over 9 percent). In contrast to the financial crisis, the period of the late 1990's is consistent with models in which growth and innovation are associated with uncertainty, e.g. due to learning, creative destruction, or a risk/return trade-off in investment projects.[2] We provide direct evidence on this point, showing that cross-sectional uncertainty and patenting activity rose and fell almost perfectly in sync during the 1990's tech boom.

Next, we examine the forecasting power of idiosyncratic uncertainty for aggregate output and employment, finding similarly mixed results. A key feature of the data is that it allows us to test whether aggregate or cross-sectional uncertainty is more relevant for forecasting, which represents a fruitful way to distinguish among classes of structural models and is also relevant for policymakers. We find strong evidence that it is aggregate rather than cross-sectional uncertainty that is most likely to be an important driver of the aggregate economy.[3]

We formally examine the cyclicality of cross-sectional uncertainty and the regressions in two theoretical models of the macroeconomic effects of cross-sectional uncertainty shocks: Christiano, Motto, and Rostagno (2014) and Bloom et al. (2018). Both models predict that cross-sectional uncertainty should be clearly countercyclical and should be more tightly related to aggregate output than aggregate uncertainty or realized volatility, which, again, is not what we observe empirically. Fully matching the data requires accommodating periods in which uncertainty is high due to good news, such as strong innovation.

In addition to evaluating correlations and forecasts, we show that the data is also useful for giving a set of moments to aid in calibrating structural models. The data series, available on our websites, gives a direct measure of the uncertainty process that needs to be parameterized in many models, showing that many papers have used realistic amounts of variation in firm-specific uncertainty, while others require implausibly high quantities.

A large literature has studied the relationship between uncertainty and the real economy. But that literature has either focused on aggregate uncertainty or, if it has looked at

[2] See, for example, Acemoglu (2005), Imbs (2007), Comin and Mulani (2009), and Kogan et al. (2017).

[3] In addition, see Berger, Dew-Becker, and Giglio (2020) for a further distinction between aggregate uncertainty and aggregate realized volatility. This is also consistent with Agarwal and Kolev (2016), who find evidence of strategic clustering of layoffs by public firms immediately after the release of aggregate bad news.

individual firms, it has not used forward-looking measures of uncertainty (like ours), but backward-looking measures (realized volatility) that do not map into what uncertainty is in models.[4] This paper shows that that distinction changes the conclusions one draws from the data. This is the first work to deliver a long time-series of *forward-looking, cross-sectional* uncertainty. Only a few papers have similar forward-looking measures of firm-level uncertainty, primarily surveys, but in those cases it is not possible to disentangle the cross-sectional and aggregate components, whereas in the case of stock returns it is straightforward.[5] This paper's novelty is in developing an ex ante measure of idiosyncratic uncertainty that more directly maps into the shock processes driving structural models and has a long empirical sample.

2 Data

We obtain options price data from the Berkeley Options Database (BODB) for 1/1980–6/1995, and Optionmetrics for 1/1996–12/2020. Appendix A.1 describes the details of the construction of the implied volatilities. Whereas the VIX is measured using a so-called model-free implied volatility, we use at-the-money Black–Scholes implied volatility. The latter requires only observing a single option price and is 99.5 percent correlated with the VIX.[6] Since implied volatilities come from asset prices, they embed risk premia, meaning they are not errorless measures of investor beliefs (see section 3.2.2).[7] In addition, these implied volatilities give uncertainty for an endogenous outcome – stock returns – rather than a fundamental shock, like TFP. Nevertheless, they represent the most common measure of uncertainty studied in the literature, and they have the attractive feature that they give a measure of uncertainty based on actual investments people have made, in addition to being forward-looking and available at high frequency.

One can always theoretically construct the linear projection of the return on stock i, $r_{i,t}$,

[4] Specifically, Campbell et al. (2001), Bloom (2009), Herskovic et al. (2016), Bloom et al. (2018), Schaal (2017), Ilut, Kehrig, and Schneider (2018), Arellano, Bai, and Kehoe (2019) and Chang, d'Avernas, and Eisfeldt (2021) all examine measures of realized dispersion rather than conditional variances. Senga (2018) studies both realized volatility and total firm implied volatility (mixing aggregate and idiosyncratic components) since 1996.

[5] See Guiso and Parigi (1999), Ben-David et al. (2013), Bachmann, Elstner, and Sims (2013), and Bachmann et al. (2018).

[6] The model-free implied volatility requires a continuum of strikes, which the available data for individual stocks does not approximate well.

[7] A constant risk premium would cause measured uncertainty to have a constant bias relative to true uncertainty. If the risk premium changes over time, but in a way perfectly correlated itself, then the volatility of measured uncertainty would also be biased, but no other properties (e.g. correlations or cyclicality) would change. Section 3.2.2 shows that fully accounting for risk premia has little impact on the results.

on the market, $r_{mkt,t}$, as

$$r_{i,t} = \alpha_{i,t} + \beta_{i,t} r_{mkt,t} + \varepsilon_{i,t} \tag{2}$$

with $\varepsilon_{i,t}$ orthogonal to $r_{mkt,t}$ by construction. (2) is just a theoretical representation – it is not directly estimable since the parameters can change on every date, nor is it structural. We follow Campbell et al. (2001) in defining cross-sectional uncertainty simply as

$$\sigma^2_{\varepsilon,t} = \sum_i w_{i,t} \sigma^2_{i,t} - \sigma^2_{mkt,t} \tag{3}$$

where $\sigma^2_{i,t}$ is a date-t conditional variance for $r_{i,t+1}$, $\sigma^2_{mkt,t}$ is the same for the market, and the $w_{i,t}$ are market capitalization weights; this equation is accurate when $\beta_{i,t} \approx 1$ (we discuss robustness to that choice below). $\sigma^2_{\varepsilon,t}$ is then the value-weighted average of residual variance – the conditional variance of $\varepsilon_{i,t+1}$ – across firms.

Since $\varepsilon_{i,t}$ is only orthogonal to the market return, it can in general be correlated across firms, e.g. due to industry effects. Changes in the volatilities of cross-sectional factors will appear in $\sigma^2_{\varepsilon,t}$ so we refer to $\sigma^2_{\varepsilon,t}$ as *cross-sectional uncertainty*.

We measure $\sigma^2_{mkt,t}$ with S&P 500 option-implied volatility, and all volatilities are interpolated to a maturity of thirty days in the main results, and 12 months in a robustness test.[8] Firm decisions naturally depend on uncertainty over more than just the next 30 days, or even probably a year. How that affects our analysis is subtle, however. We focus on the 30-day maturity for several reasons. First, because it is where options are most liquid, and it makes our results consistent with those reported in the literature using the S&P 500 VIX, which is also a 30-day measure. Second, even though the options have maturities of 30 days, the underlying stocks themselves are valued based on long-term expectations of fundamentals; so uncertainty about next month's price of a stock encodes information about the distant future. Third, if uncertainty is well approximated by an AR(1) process (e.g. Lochstoer and Muir (2021)), then short-maturity uncertainty will be a good proxy for long-maturity uncertainty. Finally, and perhaps most importantly, it is not actually clear whether shorter- or longer-term uncertainty shocks should have larger effects on investment. Hassler (1996) shows that in one tractable setting with irreversible investment, short-term uncertainty shocks have larger effects on investment than longer-term uncertainty shocks (because

[8]For the period 1980–1982, S&P 500 index options are not available. We impute values for $\sigma_{mkt,t}$ in that period with the fitted value from a regression S&P 500 implied volatility on $\sigma_{firm,t}$, S&P 500 realized volatility, the Gilchrist–Zakrajsek excess bond premium, and the S&P 500 price/earnings ratio (estimated on the period 1983–2020).

The imputation is only used for the figures and unconditional correlations. All forecasting results exclude the imputation because it involves forward-looking information.

of the interaction between the persistence of volatility, the change in the adjustment points, and the probability of reaching them). In figure A.1 in the appendix we show that for both the partial- and general-equilibrium versions of the model of Bloom et al. (2018), shorter-duration shocks also have larger effects on investment on impact (though the subsequent effects are ambiguous). Theory thus does not give a consistent answer about whether it is short- or long-term uncertainty that should be most important.

Figure A.2 in the appendix plots the fraction of total CRSP market capitalization and aggregate employment for which we have implied volatilities in each month. For the period covered by the BODB, the data covers one third of market capitalization, due to both the fact that not all firms had traded options and that only about half of those were listed on the CBOE. In 1996, when Optionmetrics becomes available, coverage by market capitalization jumps to 63 percent and then rises to 98 percent by the end of the sample. For employment, coverage rises from six-to-eight percent during the BODB sample to about 30 percent by 2020. This again reflects incomplete coverage in the early period, combined with the fact that only a minority of employment is accounted for by publicly traded firms (see also Davis et al. (2006)).

To keep the sample consistent over time, our main results calculate cross-sectional uncertainty only for the 200 largest firms in the economy over the full sample; we show below that this choice is innocuous.[9] Since we weight firms by market capitalization, and in any case only have data on public firms, our results necessarily apply to the largest firms in the economy. These firms account for a large fraction of total economic activity, though, and to the extent that idiosyncratic shocks affect the state of the economy, many theories imply it will be the largest firms whose shocks pass through to the aggregate economy (e.g. Gabaix (2011) and Acemoglu et al. (2012)).

3 Time-series behavior of cross-sectional uncertainty

This section reports the basic properties of cross-sectional uncertainty and examines its comovement with measures of real activity and financial stress.

[9]For BODB, tickers must be matched by hand to CRSP to obtain underlying stock prices. We did that only for the top 200 firms by size.

3.1 Univariate behavior and cyclicality

3.1.1 Variability

Figure 1a plots the time series of cross-sectional uncertainty, $\sigma_{\varepsilon,t}$. In the first half of the sample, there is remarkably little variation: its standard deviation is only 9 percent of its mean for the period 1980–1997. But since 1997, it rose by a factor of four to 36 percent of its mean.

		$\sigma_{mkt,t}$	$\sigma_{\varepsilon,t}$
$SD(\sigma)/E(\sigma)$:	Full sample:	0.40	0.30
	1980–1997	0.32	0.09
	1998–2020	0.43	0.36

Figure 1a also plots the implied volatility for the overall stock market. Relative to its mean, aggregate uncertainty is substantially more variable than cross-sectional uncertainty. The standard deviation of σ_{mkt} is 40 percent of its mean overall, compared to only 30 percent for cross-sectional volatility.

The variability of $\sigma_{mkt,t}$ is also much less isolated in time. Whereas the variation in cross-sectional uncertainty is driven primarily by just three episodes, there are numerous substantial jumps in market-level uncertainty, associated with the 1987 stock market crash, the first Gulf War, various events between 1998 and 2002, the debt ceiling, the Euro crisis, etc.

The relative volatilities of $\sigma^2_{\varepsilon,t}$ and $\sigma^2_{mkt,t}$ can be used to construct a variance decomposition for the total variance faced by firms. Specifically,

$$\begin{aligned}
\operatorname{var}\left(\textstyle\sum_i w_{i,t}\sigma^2_{i,t}\right) &= \operatorname{var}\left(\sigma^2_{mkt,t}\right) + \operatorname{var}\left(\sigma^2_{\varepsilon,t}\right) + 2\operatorname{cov}\left(\sigma^2_{i,t},\sigma^2_{mkt,t}\right) \\
4.3\times10^{-3} &= \underset{27\%}{1.2\times10^{-3}} + \underset{47\%}{2.0\times10^{-3}} + \underset{26\%}{1.1\times10^{-3}}
\end{aligned}$$

Over the sample, the variation in the total uncertainty that firms face is relatively more driven by variation in idiosyncratic than aggregate uncertainty. In addition, the fact that idiosyncratic and aggregate uncertainty are correlated also matters, accounting for about one quarter of the total variation in firm uncertainty.

The moments reported here on the volatilities of aggregate and cross-sectional uncertainty are useful for calibrating structural models of uncertainty shocks. We return to this point below.

3.1.2 Cyclicality

Figure 1b plots cross-sectional uncertainty against the linearly detrended level of the CRSP total stock market index. The periods of high cross-sectional uncertainty are all associated with large changes in stock prices, but in different directions. During the dot-com boom, cross-sectional uncertainty tracks the rise of the stock market. They peak in almost exactly the same month, and uncertainty declines with the market. It follows the opposite pattern during the financial crisis and coronavirus episodes: it is exactly when the stock market declines that cross-sectional uncertainty rises (though Covid is a bit different in that the stock market quickly recovered while cross-sectional uncertainty remains high). So uncertainty appears to be procyclical in the late 1990s and early 2000s, countercyclical in the financial crisis and in the recent coronavirus episode, and acyclical otherwise.

Figures 1c and 1d further emphasize that point by plotting cross-sectional uncertainty against aggregate investment and the unemployment rate. Investment and uncertainty peak simultaneously in 2000, while uncertainty spikes and investment crashes in both 2008 and 2020. Figure 1d shows that uncertainty has a similarly mixed relationship with the unemployment rate.

To more formally quantify the cyclicality of cross-sectional uncertainty, panel a of table 1 reports the correlation of cross-sectional uncertainty with various measures of the state of the economy, over the full sample, pre-2020, and pre- and post-1/2008. We choose the 2008 break point because it is where, from the figure, uncertainty becomes clearly countercyclical – i.e. after the dot-com crash. The series labeled as detrended are HP-filtered with the usual monthly parameter of 129,600 (table A.1 in the appendix examines robustness to alternative detrending choices).

In terms of levels, cross-sectional uncertainty does not have a consistent correlation with economic indicators. It is weakly positively correlated with the CBO output gap, detrended employment and minus the unemployment rate, implying it is procyclical. Its correlations with detrended industrial production and capacity utilization negative but close to zero, and its correlation with credit spreads is positive, indirectly implying it is countercyclical. So relative to levels, it appears essentially acyclical. Furthermore, most of those correlations change sign between the first and second parts of the sample. Relative to growth rates, uncertainty appears more consistently countercyclical, with negative correlations with all the cyclical measures, though the coefficients are generally small. However, when 2020 is excluded, the correlations with the growth rates become significantly stronger.

Comparing uncertainty and unemployment, there are two ways to look at the data. First, of the three peaks in uncertainty, two are associated with high unemployment and one low.

Second, across the six peaks in unemployment, uncertainty is high in two, low in one, and near its average in the other three.

So in terms of simple correlations, the link between cross-sectional uncertainty and the business cycle is weak, with mainly small correlations that change sign across measures and subsamples.

3.1.3 Uncertainty and innovation

A natural explanation for the fact that uncertainty is high sometimes in expansions and sometimes in recessions is that in some periods uncertainty is associated with growth and innovation, in particular during the tech boom. That innovation might be associated with creative destruction and reallocation across firms. In the next section we examine the extent to which overall cross-sectional uncertainty is driven by volatility in the tech sector itself (the answer is very little), but before doing that, here we just ask whether uncertainty is related to overall innovation in the economy.

Figure 1e plots cross-sectional uncertainty compared to the index of the total value of patents relative to GDP of Kogan et al. (2017). The peak in uncertainty in the late 1990's almost perfectly tracks, both on the way up and the way down, the peak in patents relative to GDP. The fit here is much better than for the S&P 500 VIX, showing that it is really cross-sectional rather than aggregate uncertainty that was tightly linked to innovation in that period. The second major rise in uncertainty, in the financial crisis, is significantly different, with only a small rise in patents, which came before the peak in uncertainty.

Figure 1e therefore shows clearly the difference between the two first two peaks in cross-sectional uncertainty and why they might have been associated with different economic outcomes. One was associated – with almost identical timing – with a huge run-up in patenting activity and hence innovation, while the other was associated with no increase in patenting at all (and, instead, with a financial crisis).

3.2 Robustness and further results

This section examines the sensitivity of the results above to a range of perturbations of the analysis. We begin by using sector-level variation in uncertainty and employment to examine the relationship at a more granular level, and then examine a number of modifications of the measurement of uncertainty relative to the baseline case.

3.2.1 Sector analysis

Since we have uncertainty at the firm level, instead of aggregating across all firms, we can also aggregate at the sector level. We define cross-sectional uncertainty in sector j as

$$\sigma^2_{\varepsilon,j,t} = \sum_{i \in j} w_{i,t}\sigma^2_{i,t} - \sigma^2_{mkt,t} \tag{4}$$

The advantage of sector-level uncertainty is that it allows us to examine the relationship between uncertainty and activity after controlling for sector and time fixed effects, asking whether sectors that have uncertainty higher than average (their own average and the average in a given month) have lower output.

We calculate cross-sectional uncertainty at the two-digit NAICS sector level and compare it to employment in the same sector from the BLS. There are sufficient stocks for us to be able to create a balanced panel for 15 two-digit sectors. We estimate simple regressions of the form

$$\Delta \log Empl_{j,t} = a_j + b_t + c\sigma_{\varepsilon,j,t} + \eta_{j,t} \tag{5}$$

where $Empl_{j,t}$ is employment in sector j and month t, a_j and b_t are sector and time fixed effects, respectively, c is a coefficient, and $\eta_{j,t}$ is a residual, which we cluster by sector.[10] We also normalize $\Delta \log Empl_{j,t}$ and $\sigma_{\varepsilon,j,t}$ to have unit standard deviations after controlling for time and sector fixed effects, so that the coefficient c can be interpreted as a correlation.[11]

We estimate three versions of the regression, the first using $\Delta \log Empl_{j,t}$ and $\sigma_{\varepsilon,j,t}$ directly, the second removing a HP-filtered trend (calculated sector-by-sector) from $\Delta \log Empl_{j,t}$, and the third using the HP-filtered $\log Empl_{j,t}$. The coefficient c and its 90-percent confidence band are reported below for the three different regressions:

Dependent variable	Coefficient, 90% CI
$\Delta \log Empl_{j,t}$	-0.03 $[-0.06,-0.00]$
HP filtered $\Delta \log Empl_{j,t}$	-0.09 $[-0.19,0.00]$
HP filtered $\log Empl_{j,t}$	0.04 $[-0.01,0.08]$

As in table 1a, the coefficients are small with mixed signs. The confidence bands are reasonably narrow in economic terms, but the coefficients are small enough that they are still only

[10] We examined various estimation methods to account for the small number of clusters (e.g. bootstrapping schemes) and they had little quantitative effect on the results.

[11] Formally, we regress $\Delta \log Empl_{j,t}$ and $\sigma_{\varepsilon,j,t}$ on time and sector dummies (a_j and b_t), and define $\Delta \log \widehat{Empl}_{j,t}$ and $\widehat{\sigma_{\varepsilon,j,t}}$ to be the residuals from those regressions. We then set $\Delta \log \widetilde{Empl}_{j,t} = \Delta \log \widehat{Empl}_{j,t} / stdev\left(\Delta \log \widehat{Empl}_{j,t}\right)$ and regress $\Delta \log \widetilde{Empl}_{j,t}$ on $\widetilde{\sigma_{\varepsilon,j,t}}$.

barely significant at the 10-percent level. Again, they can be interpreted as correlations, so they say that, at the point estimates (and after controlling for time and sector fixed effects), the sector-level correlation between uncertainty and unemployment is less than 10 percent. The conclusions from examining cross-sectional variation thus reinforce those from table 1a.

3.2.2 Time-varying risk premia

So far we have ignored the presence of time-varying risk premia, assuming implicitly that $\sigma^2_{\varepsilon,t}$ is perfectly correlated with the true conditional standard deviation of firm-specific residuals, $E_t\left[\sum_i w_{i,t}\varepsilon^2_{i,t+1}\right]$. That is, we do not necessarily require $\sigma^2_{\varepsilon,t}$ to be the physical (objective) conditional standard deviation; rather, so far we have allowed a constant risk premium on cross-sectional variance, or a risk premium that is affine in $E_t\left[\sum_i w_{i,t}\varepsilon^2_{i,t+1}\right]$. However, it is entirely possible that the risk premium depends on other factors, such as the state of the business cycle.

If we take the view that risk premia might depend on some set of state variables $\{x_{j,t}\}$, such as, e.g., the unemployment rate, then it is straightforward to show that true cross-sectional uncertainty can be recovered from a projection of $\sum_i w_{i,t}\varepsilon^2_{i,t+1}$ on the date-t state variables (see appendix A.3). We estimate the regression

$$\sum_i w_{i,t}\varepsilon^2_{i,t+1} = b_0 + b_\sigma \sigma^2_{\varepsilon,t} + \sum_j b_j x_{j,t} + \eta_t \tag{6}$$

where the b's are coefficients, the x_j's are state variables, and η_t is a residual. The uncertainty measure that is robust to time-varying risk premia is then the fitted value from that regression. For state variables, we include the date-t value and first lag of realized cross-sectional dispersion $\left(\sum_i w_{i,t-1}\varepsilon^2_{i,t}\right)$, lagged option-implied uncertainty $(\sigma^2_{\varepsilon,t-1})$, the unemployment rate, the Gilchrist–Zakrajsek credit spread, and the S&P 500 price/earnings ratio.

Figure 2a plots the baseline cross-sectional uncertainty measure, $\sigma_{\varepsilon,t}$, against the version that is robust to risk premia (the square root of the fitted value from the regression (6)). They are 98 percent correlated, and the figure shows that their behavior is economically nearly identical. That result holds because the dominant driver of the regression (6) is in fact $\sigma^2_{\varepsilon,t}$ – it has the largest t-statistic and hence largest marginal R^2 of all the variables. While is it true that other variables show up as statistically significant in the regression, indicating that $\sigma^2_{\varepsilon,t}$ is not completely free of time-varying risk premia, as an economic matter it appears to be close enough that our conclusions are unchanged once risk premia are more formally accounted for.

3.2.3 Accounting for industry effects

We account for industry effects in the analysis in two ways. The first focuses on the tech sector, as it may have been a particularly large contributor, at least to the late-1990's volatility, and possibly also later, while the second accounts for industry exposures more generally. Whereas section 3.2.1 calculated cross-sectional uncertainty in each sector, this section calculates an average cross-sectional uncertainty controlling for sector-level shocks. Those are subtly different exercises, as will be made clear in the equations that follow.

The method we use to account for volatility in the tech sector is general and could be applied to any potential cross-sectional factor. Suppose stock returns have the factor structure,

$$r_{i,t} = \beta_i r_{mkt,t} + \gamma_i \hat{r}_{tech,t} + \varepsilon_{i,t} \tag{7}$$

where $r_{mkt,t}$ is again the return on the overall market, and $\hat{r}_{tech,t}$ is the value-weighted return on the tech sector (which we take as stocks in the GICS 45 sector) orthogonalized with respect to $r_{mkt,t}$, which just has the effect of normalizing the loadings such that β_i has a value-weighted mean of 1 and γ_i has a value-weighted mean of 0 (across the full universe of stocks).

If we have an uncertainty measure $\sigma^2_{tech,t} = E_t\left[\hat{r}^2_{tech,t+1}\right]$, then, based on the orthogonality discussed above, we have:

$$\sigma^2_{\varepsilon,t} = \left(\sum_i w_{i,t}\sigma^2_{i,t}\right) - \left(\sum_i w_{i,t}\beta_i^2\right)\sigma^2_{mkt,t} - \left(\sum_i w_{i,t}\gamma_i^2\right)\sigma^2_{tech,t} \tag{8}$$

We calculate $\hat{r}_{tech,t}$ by taking the value-weighted return on stocks in the GICS 45 sector, orthogonalized with respect to the market return. The coefficients β_i and γ_i can then be estimated from firm-level regressions. Options on the tech sector are not available until relatively late in the sample, so for this exercise we calculate $\sigma^2_{tech,t}$ by forecasting $\hat{r}^2_{tech,t}$ using its own lagged values, as in a GARCH model (formally, we use a so-called heterogeneous autoregressive specification, including squared returns over the previous week, month, and quarter).

Figure 2b plots $\sigma_{\varepsilon,t}$ under the baseline case and after accounting for exposure to a tech sector factor. The time series are again highly similar. The quantitatively small effects are due to two factors. First, $\sigma^2_{tech,t}$ is generally relatively small – its volatility is only 30 percent of the volatility of $\sigma^2_{mkt,t}$. Second, since γ_i has a cross-sectional mean of zero, $\sum_i \omega_{i,t}\gamma_i^2$ represents its variance. That variance is also in general small, averaging only 0.38, which causes the tech factor to ultimately have quantitatively small effects.

The second way that we account for industry effects is to control for the industry that each firm is in. In particular, if we assume for simplicity that each firm's exposure to its own industry factor is 1 (in the same way that the baseline results assume that exposures to the market return are 1), then an industry-robust estimate of cross-sectional uncertainty is

$$\sigma^2_{\varepsilon,i,t} = \sigma^2_{i,t} - \sigma^2_{ind(i),t} \tag{9}$$

In this case, we calculate $\sigma^2_{ind(i),t}$ as the implied variance for the SPDR exchange traded fund covering stock i's sector. Figure 2b also plots that series relative to the baseline. The main differences are again that cross-sectional uncertainty is somewhat dampened in both the tech boom and financial crisis, consistent with the idea that those two episodes were driven primarily by sector shocks (to tech and finance, respectively).

3.2.4 Additional robustness tests

Panels c–f of figure 2 plot five variations on the benchmark uncertainty series:

1. Using the median of implied volatility across firms (after taking out firm fixed effects), instead of weighting by market capitalization (panel c).

2. Using the full sample of options from Optionmetrics instead of just the largest 200 firms (also in panel c).

3. Correcting for each firm's loading on the market, by estimating β_i for each firm and setting $\sigma^2_{\varepsilon,i,t} = \sigma^2_{i,t} - \beta_i^2 \sigma^2_{mkt,t}$ (panel d).

4. Weighting by employment instead of market capitalization (panel e).

5. 12-month uncertainty (panel f).

The first test shows that the results are not driven just by the weighting by market capitalization – cross-sectional median uncertainty displays similar behavior. The main difference is a smaller increase in the late 1990's.

Second, if we use the full Optionmetrics sample instead of just the 200 largest firms, the results are essentially unchanged, with just a level shift. That result is consistent with the largest firms having relatively lower uncertainty overall, and it suggests that if we could measure uncertainty for all firms in the economy, it would be higher. However, the dynamics of uncertainty when including smaller public firms are nearly identical to those of just the largest firms, implying the presence of a strong common factor affecting all firms, big and small.

The third test shows that the approximation where we treat the loadings on the market as all equal to 1, as in Campbell et al. (2001), has very little impact.

The fourth test – weighting by Compustat employment – is novel to this paper. Past work, in focusing on the S&P 500 index, puts weight on firms and their volatility based on their equity valuation. Differences in leverage and investor beliefs will then affect how uncertainty is aggregated, whereas employment weighting gives a more stable and fundamental measure. The results turn out to be highly similar. The largest effect is during the tech bubble, when tech stocks had high market capitalization relative to their share of the real economy and also relatively high implied volatilities.

Finally, we also plot cross-sectional uncertainty measured from 12-month instead of 1-month options (using only the Optionmetrics sample, which has better coverage of long maturities). The level of the time series is shifted up slightly (consistent with the presence of a small risk premium), but it otherwise nearly identical to the benchmark case, with a correlation of 98 percent in levels and 88 percent in quarterly changes. The high correlation is consistent with the results from Lochstoer and Muir (2021), discussed above, that an AR(1) process is a good approximation for uncertainty empirically.

To get an additional comparison for option-implied uncertainty, we examined the Federal Reserve Bank of Atlanta's Survey of Business Uncertainty (SBU), which is available since 2016. Figure A.3 plots sales and employment uncertainty from that survey compared to our option-implied measure. All three series have little variation between 2016 and the end of 2019, then rise by about 50 percent on average in 2020. The relative magnitude of the increase across the three series is very similar, and they all remained high through 2020. So while the comparison only covers a single event, the option-implied measure is validated by the SBU measure.

Appendix A.2 discusses the commonality in variation in uncertainty across firms. Consistent with Herskovic et al. (2017), 40–50 percent of cross-sectional variation in firm-specific uncertainty is captured by the common component.

Finally, we have also constructed firm-specific uncertainty using the so-called model-free implied volatility and obtain nearly identical results.

3.3 Uncertainty versus realized dispersion

Past studies looking at cross-sectional uncertainty have used realized dispersion – the cross-sectional standard deviation of the realizations of $\varepsilon_{i,t}$ – to proxy for $\sigma^2_{\varepsilon,t}$ and extend the sample to earlier periods (e.g. Davis et al. (2006) and Bloom (2009)). The difference between the two is not innocuous.

Figure 1f plots $\sigma^2_{\varepsilon,t}$ against the realized cross-sectional standard deviation of the firm-specific residuals, $\varepsilon_{i,t}$. The realized standard deviation behaves significantly differently from the conditional standard deviation, $\sigma_{\varepsilon,t}$, appearing to have substantial high-frequency "noise".

Recall that $\sigma^2_{\varepsilon,t-1}$ is the conditional expectation of the realized dispersion in returns, defined as

$$RD_t^{ret} = \sum_i w_{i,t} \operatorname{var}(r_{i,t}) - \operatorname{var}(r_{mkt,t}) \tag{10}$$

where $\operatorname{var}(r_{i,t})$ and $\operatorname{var}(r_{mkt,t})$ are calculated from *realized* returns within each month. Therefore,

$$RD_t^{ret} = \sigma_{\varepsilon,t-1} + \eta_t \tag{11}$$

where η_t is a mean-zero shock uncorrelated with $\sigma_{\varepsilon,t-1}$.[12]

Equation (11) means that if one's goal is to understand the behavior of *uncertainty* (the forward-looking $\sigma_{\varepsilon,t-1}$) – its variability, correlation with other variables, or its coefficient in forecasting regressions – then proxying for it with RD_t^{ret} will cause biases. The volatility of RD_t^{ret} is substantially higher than that of $\sigma_{\varepsilon,t-1}$, its correlation with other variables is lower, and in regressions there will be an attenuation bias even if η_t is exogenous. If η_t is correlated with outcomes of interest, that will further bias any regressions. Berger, Dew-Becker, and Giglio (2020), for example, show that when structural productivity shocks are skewed left (consistent with observed asymmetry in the business cycle), then realized volatility, η_t, will be negatively correlated with output, even if there is no structural effect of uncertainty on activity.

Furthermore, for a policymaker working in real time, the relative precision of $\sigma_{\varepsilon,t}$ is an added advantage. If one's goal is to measure uncertainty in real time, the added noise in realized dispersion makes it less useful than the true uncertainty $\sigma_{\varepsilon,t}$. To see the difference between the two series, the table below reports the pairwise correlation between $\sigma_{\varepsilon,t}$ and RD_t^{ret} for levels, monthly changes, and quarterly changes.

Correlations between $\sigma_{\varepsilon,t}$ and RD_t^{ret}:

Levels	0.92
Monthly changes	0.66
Quarterly changes	0.80

In terms of levels, they are 92 percent correlated, meaning that it will be difficult to

[12] Formally, this is true up to a first-order approximation, and holds literally for variances rather than standard deviations. However, the qualitative behavior of the series is the same with and without the square root transformation.

disentangle them in many cases. In differences, though, the correlations are far smaller – 0.66 and 0.80 for monthly and quarterly changes, respectively. A way to quantify the noise in $\sigma_{\varepsilon,t}$ and RD_t^{ret} is to calculate the autocorrelation of their first differences. That is -0.01 for $\sigma_{\varepsilon,t}$, but -0.31 for RD_t^{ret}, indicating that the latter has larger transitory variation. We show below that this translates into $\sigma_{\varepsilon,t}$ being relatively more useful for forecasting future dispersion.

4 Forecasting

This section examines the ability of cross-sectional uncertainty to forecast both future realized cross-sectional dispersion and also aggregate real activity.

4.1 Realized dispersion

The first question one must ask about the forecasting power of option-implied uncertainty is whether it forecasts future realized dispersion, as predicted by equation (11). Does it actually measure uncertainty?

Panel b of table 1 reports results of regressions of quarterly realized dispersion on the lag of $\sigma_{\varepsilon,t}$. We report 90-percent confidence intervals in brackets below (none of the main results are sensitive to the choice of a 90- versus 95-percent cutoff, though some auxiliary results are). In all cases, both the dependent and independent variables are standardized to have unit standard deviations. The first two columns show that $\sigma_{\varepsilon,t}$ has substantial forecasting power for RD_{t+1}^{ret}. A unit standard deviation increase in cross-sectional uncertainty is associated with future cross-sectional uncertainty higher by 0.89 standard deviations, indicating that cross-sectional uncertainty is close to an unbiased predictor of changes in realized dispersion over time (the result is similar in absolute levels instead of standard deviations).[13] That significant predictive power remains even after controlling for lagged realized volatility (though the confidence bands are wide enough that the coefficients are not statistically significantly different from each other).[14]

The second and third pairs of columns of table 1b report results for two alternative measures of cross-sectional dispersion: the cross-sectional interquartile ranges of growth in industrial production (across sectors) and growth in sales (across Compustat firms), RD^{IP}

[13] The constant in the regression is not equal to zero, consistent with the presence of a risk premium.

[14] See Lochstoer and Muir (2021) for a more detailed analysis of the the predictive power of option prices for *aggregate* as opposed to cross-sectional volatility.

and RD^{sales}.[15] In both cases, $\sigma_{\varepsilon,t}$ again has substantial forecasting power. A unit standard deviation increase in uncertainty predicts about a 0.2 standard deviation increase in future dispersion. The relatively smaller magnitude is not surprising since $\sigma_{\varepsilon,t}$ measures uncertainty of stock returns, rather than IP or sales. In the case of IP, which is available at the monthly level, the significant predictive power survives (at the 90- but not 95-percent level) even after controlling for lagged RD^{IP}, though the coefficient shrinks substantially.

To the extent that $\sigma_{\varepsilon,t}$ has predictive power, a natural question is whether it comes through $\sigma_{\varepsilon,t}$ or RD^{ret}, given how strongly correlated they are. That correlation makes them rather difficult to separately identify. The results in table 1b for forecasting RD^{ret} find a larger coefficient on $\sigma_{\varepsilon,t}$, but again the coefficients are not significantly different from each other. Table A.2 in the appendix shows a similar result holds in forecasting realized dispersion in IP and sales growth.

4.2 Real activity

We study forecasts of three monthly variables: the unemployment rate, non-farm private employment growth, and industrial production growth. All are again standardized to have unit variance. These regressions are valuable for two reasons, both independent interest and also as moments that can be used to test models. These regressions stop at the end of 2019 because the shifts in 2020 are so large as to be dominant in the data (the one-month growth rates are ten times larger than anything prior). However, the results are qualitatively consistent using the sample through 2020.

The first column in the three sections of table 1c reports a regression of activity on lagged cross-sectional uncertainty. In all three cases, the coefficient implies that increases in uncertainty are followed by declines in real activity. The magnitudes of the coefficients are similar, with a unit standard deviation increase in uncertainty being associated with declines in IP and employment of about 0.13 standard deviations and an increase in unemployment of 0.22 standard deviations.

As in the previous section, though, these results should be interpreted with caution, as they appear to be sensitive to the sample used. Furthermore, we note that there is no claim of any sort of identification of shocks here. These are simple one-step-ahead forecasting regressions. So while one interpretation is that they imply that uncertainty causes declines in activity, another is that there is some other factor driving both activity and uncertainty,

[15] As in Bloom et al. (2018), we study the interquartile range because IP and sales growth have heavy tails, causing the cross-sectional standard deviations to be driven by outliers. Table A.2 in the appendix reports results for the cross-sectional standard deviations.

and that uncertainty just responds relatively quickly.

Sharper tests of theoretical models can be obtained by contrasting the forecasting power of cross-sectional uncertainty with that of other measures, like realized cross-sectional dispersion and aggregate uncertainty. Empirically, these measures are correlated, which makes identification difficult, but the existing literature has often offered sharp predictions about which type of volatility matters for the real economy. The correlation between cross-sectional uncertainty and realized dispersion is 0.92, while the correlation between cross-sectional and aggregate uncertainty is 0.56, meaning that the latter pair of variables should be much easier to distinguish.

Studying the effects of *aggregate* uncertainty, Berger, Dew-Becker, and Giglio (2020) show that uncertainty is driven out by aggregate realized volatility (the aggregate analog to realized dispersion). The second column of each section of table 1c shows that for this paper's analysis, when *cross-sectional* realized dispersion is included in the forecasting regressions, it is associated with declines in employment and increases in unemployment, but it has no effect on industrial production growth. That said, the (90-percent) confidence bands in these regressions are wide due to the high correlation between implied and realized dispersion, so that it is difficult to draw clear conclusions, except to say that there is not strong evidence here that uncertainty has significantly negative effects on its own.

Those results, together with those in the previous section, tell us that even though forward-looking cross-sectional uncertainty $\sigma_{\varepsilon,t}$ is an important predictor for realized dispersion – showing up as significant even after controlling for lagged realized dispersion – it is relatively weakly associated with real activity, whether lagged dispersion is included as a control or not.

The last column of each section of table 1c asks whether cross-sectional or aggregate uncertainty is more important for forecasting. In all three cases, market uncertainty drives cross-sectional uncertainty out of the regressions. Not only is market uncertainty dominant in relative terms, its coefficients are also large in absolute terms, -0.21 for employment and IP, and 0.25 for unemployment, and they are highly statistically significant. The data therefore suggests that to the extent that uncertainty is relevant for forecasting real activity, it is aggregate rather than cross-sectional uncertainty that matters.[16]

Overall, while cross-sectional uncertainty does have some univariate forecasting power, it is delicate; there is evidence that realized volatility may be equally important, and market uncertainty seems to dominate idiosyncratic uncertainty. But, again, the forecasting results

[16] Though, again, see Berger, Dew-Becker, and Giglio (2020). The question of whether aggregate uncertainty is an exogenous driving force or endogenous to the state of the business cycle is also studied by Ludvigson, Ma, and Ng (2021) and Ferrer, Rogers, and Xu (2021).

are statistically noisy, underscoring the fact that the relationship between cross-sectional uncertainty and the business cycle is simply not very strong, regardless of what direction it runs on average and which measure is dominant.

4.3 Summary

Section 3 showed that over the last 40 years, cross-sectional uncertainty has been relatively stable outside of three distinct episodes. Of those three peaks, two came during recessions and one during an expansion – and tightly associated with a huge boom in patenting activity – showing that cross-sectional uncertainty has had a mixed relationship with the business cycle. This section takes that result a step further, showing that cross-sectional uncertainty overall has weak forecasting power for real activity. The results are difficult to reconcile with the view that firm-specific uncertainty is, in and of itself, a major economic headwind. Sometimes it may be a consequence of things actually going well.

Two alternative variables, realized dispersion and aggregate uncertainty, drive cross-sectional uncertainty out of the forecasting regressions we run. Between aggregate and cross-sectional uncertainty, aggregate uncertainty appears to be the relevant driver. Furthermore, to the extent that the cross-section matters, it is through the realization of shocks that generate dispersion or reallocation, not the expectation that such shocks might occur (even though realized dispersion, to the naked eye, simply appears to equal ex ante uncertainty plus significant noise).

The next section shows how these results can be used to calibrate and test structural models.

5 Calibrating and testing structural models

We now use $\sigma_{\varepsilon,t}$ to calibrate and test structural models. We first estimate the time-series dynamics of uncertainty, a key input to calibrations. The regressions from table 1c are a useful test of structural models. Since most models are constructed at the quarterly frequency, the empirical results reported in this section are also estimated at the quarterly frequency.

We begin in section 5.1 by showing how moments from our dataset can be used to calibrate models. Sections 5.2 and 5.3 then focus on testing two specific models: the "really uncertain business cycles" (RUBC) model of Bloom et al. (2018), which is centered around a real options framework, and the model of financial frictions of Christiano, Motto, and Rostagno (CMR; 2016).

5.1 Calibration moments

In the vast majority of models with time-varying uncertainty, uncertainty follows an AR(1) type process, which can be characterized by its standard deviation (which we scale relative to its mean) and autocorrelation. Table 2a reports those moments in the data, along with bootstrapped 95-percent confidence bands. These numbers are useful both as a guide for future calibrations and also for evaluating calibrations used in past work. As discussed above, the standard deviation of cross-sectional uncertainty relative to its mean is 30 percent over the full sample. Its quarterly autocorrelation is 0.91.

The next five columns of the table report analogous population moments in the calibrations of recent structural models. In all five cases, the idiosyncratic uncertainty represents the volatility of firm-specific shocks to fundamentals (typically productivity). For the scaled standard deviation, the calibrations range from 0.09 to 0.71, lying inside the empirical confidence band in only a single case. For the autocorrelation, the calibrations range from 0.71 to 0.98 and are inside the empirical confidence bands in four of five cases. The table shows that there is little agreement in the literature on either the volatility or persistence of idiosyncratic risk. The data presented here can help resolve that disagreement.

5.2 Correlations

Table 2b reports raw correlations between major economic aggregates and uncertainty at the quarterly frequency. As in table 1, the correlations in the data are weak and have mixed signs. The second and third columns report population correlations in the RUBC and CMR models. In both cases, with the exception of consumption growth, the correlations are substantially negative and well outside the empirical confidence bands. Both models focus on contractionary effects of uncertainty shocks. While the data presented here is consistent with the existence of such mechanisms during some episodes, in that uncertainty was high during the 2009 and 2020 recessions, the large increase in cross-sectional uncertainty during the boom of the late 1990's renders the overall correlation close to zero.

5.3 Regressions

A third way that the data can be used to evaluate models is to ask whether they can generate behavior similar to the regression results obtained in table 1c. Recall that in addition to finding that cross-sectional uncertainty alone had some forecasting power, we also found that it was driven out by both realized cross-sectional dispersion and aggregate uncertainty. The models, particularly RUBC, are not necessarily meant to match all the dynamics of the economy, so here we study a relatively simple specification, which is to regress changes in growth rates of macro aggregates on changes in implied and realized volatility, thus avoiding issues of lag lengths, overshooting, etc.

To help focus in particular on the effects of uncertainty *shocks* (though certainly not formally identified shocks), in both the model and data here we regress changes in macro aggregates on changes in uncertainty and innovations to realized dispersion. For CMR, then, the regression we estimate in the data and model is

$$\Delta y_t = b_0 + b_1 \Delta y_{t-1} + b_2 \Delta \sigma_{\varepsilon,t} + b_3 \Delta^{IV} RD_t^{ret} + \eta_t \tag{12}$$

where η_t is a residual, y represents the log of GDP, consumption, investment, or hours worked, and Δ denotes the first-difference operator. For RD_t^{ret}, we set $\Delta^{IV} RD_t^{ret} \equiv RD_t^{ret} - \sigma_{\varepsilon,t-1}$, which represents the surprise in realized dispersion (since $\sigma_{\varepsilon,t-1}$ is its expectation on date $t-1$).[17] As in the analysis above, all variables are standardized to have unit variance. The model of CMR is sufficiently rich that it includes stock returns. $\sigma_{\varepsilon,t}$ is thus measured in the model as the conditional standard deviation of the firm-specific component of stock returns – that is, the concept in the model explicitly matches what is measured empirically.

The first column in table 2c reports the estimates from the data, while the second column reports the (population) coefficients in CMR. In all four cases, increases in uncertainty are actually associated with *increases* in activity, while realized dispersion has a consistently negative relationship with real activity. In simulations of the CMR model, on the other hand, in all four cases it is shocks to uncertainty that are most important for explaining real activity, rather than realized dispersion. As discussed above, realized dispersion is equal to uncertainty plus noise (the unexpected component of realized dispersion). In the CMR model, that noise does not have structural effects, so uncertainty dominates the regressions.

In the data, though, that "noise" – the gap between realized dispersion and its expectation – actually contains information. Models featuring concave responses to shocks, such as Ilut, Kehrig, and Schneider (2018) and Dew-Becker, Tahbaz-Salehi, and Vedolin (2020), are able to generate that effect. That is, in those models, realized dispersion does have effects on output, above and beyond the expected component encoded in uncertainty.

As discussed above, given that with our data we can construct a pair of matching measures of both aggregate and cross-sectional uncertainty, we can address the question of which of the two is more important for driving fluctuations. While CMR only has time-varying cross-sectional uncertainty, RUBC has fluctuations in both cross-sectional and aggregate

[17]the results are similar, though slightly weaker and less consistent, when we replace $\Delta^{IV} RD_t^{ret}$ with the simple difference ΔRD_t^{ret} (sdue to the large transitory component in RD_t^{ret} visible in figure 1f.

uncertainty, so we want to test, both in the data and the model, which is more important.

To do so, the second pair of columns in table 2c reports estimates from a version of (12) where we replace RD_t^{ret} with $\sigma_{mkt,t}$,

$$\Delta y_t = b_0 + b_1 \Delta y_{t-1} + b_2 \Delta \sigma_{\varepsilon,t} + b_3 \Delta \sigma_{mkt,t} + \eta_t \tag{13}$$

That regression then allows us to measure the relative importance of cross-sectional and aggregate uncertainty. Unlike in CMR, the RUBC model does not explicitly model stock returns. $\sigma_{\varepsilon,t}$ and $\sigma_{mkt,t}$ are thus calculated in the model as just the conditional volatilities of the firm-specific and aggregate components of technology shocks, thus making the connection between the model and data a bit weaker than for CMR.

As in table 1c, table 2c shows that aggregate uncertainty drives cross-sectional uncertainty out of the empirical forecasting regressions. GDP, consumption, investment, and hours are all substantially more strongly driven by shocks to aggregate than cross-sectional uncertainty, with the coefficients on cross-sectional uncertainty actually being positive in each case.

Table 2c shows that the coefficients on aggregate uncertainty in the forecasting regression run on simulations of the RUBC model are, instead of being dominant, in all cases except consumption (which responds positively to both types of uncertainty) much smaller than the coefficients on cross-sectional uncertainty, by factors of three to four.[18] In the RUBC model, it is primarily variation in cross-sectional rather than aggregate uncertainty that matters. That fact makes sense given that in the model, the vast majority of the variation in the total uncertainty faced by firms is from the cross-sectional component, but it is the opposite of what is observed empirically.

5.4 Implications

Table 2 makes two basic contributions. The top panel gives specific moments – volatility and autocorrelation – for calibrating structural models. The second contribution is to show how the raw correlations and regressions provide insights into aspects of the data that models can and cannot match. The data implies that realized dispersion and aggregate uncertainty both drive cross-sectional uncertainty out of the regressions. Both of those features of the

[18] In the benchmark model in the RUBC paper, cross-sectional and aggregate uncertainty are perfectly correlated. We run the simulation code three different times with calibrations where cross-sectional and aggregate uncertainty vary by different amounts, so that when the three calibrations are combined, the perfect correlation is broken. In the baseline RUBC calibration, aggregate and cross-sectional uncertainty rise by a factors of 1.61 and 4.14, respectively, in the high-uncertainty state. We construct two additional simulations in which they rise by the factors $\{2.42, 2.76\}$ and $\{1.07, 6.21\}$ and append them to the baseline simulation.

data are difficult for two leading models to match, so future work could use them as areas for improvement.

As discussed above, the targets in panels b and c of table 2 are difficult to match within a single model – they require being able to match the fact that uncertainty is sometimes good and sometimes bad, and also being able to explain why realized dispersion would have independent effects (which may require a nonlinear model). It is no criticism of RUBC and CMR that their benchmark calibrations do not match these new results. Rather, table 2 simply yields new empirical facts that structural models can be built or enriched to match.

6 International evidence

To explore the behavior of cross-sectional uncertainty internationally, we obtain data from Optionmetrics Europe for 1/2002 to 12/2018 and from Bloomberg for 1/2019 to 12/2020. We have acceptable data for Switzerland, Germany, France, Great Britain, the Netherlands, and the constituents of the Euro Stoxx 50 index. While all of the countries are from western Europe, the list includes countries with varying degrees of connection to the EU and countries on different currencies and with very different government fiscal states.

Figure 3 plots cross-sectional uncertainty for each country against the US. In each case, cross-sectional uncertainty is clearly strongly correlated with that in the US, elevated in 2002, declining until the financial crisis, and then low and stable from 2010 until coronavirus in 2020 (the correlations are reported in the figure). That is true even though the path of aggregate output in Europe over this period was very different from the US – a number of these countries went into recessions around 2012.

The table below summarizes the time-series behavior of market and cross-sectional uncertainty in the international data. The average time series standard deviation of market uncertainty is nearly twice that of cross-sectional uncertainty. Furthermore, it confirms the results on the fraction of the variation in uncertainty explained by a common factor – this time using the cross-sectional mean of uncertainty, rather than the US value. Finally, it reports the simple average of all the pairwise correlations across countries and shows the high degree of similarity for both types of uncertainty.

Statistics for cross-sectional and market uncertainty across countries

	Cross-sectional unc.	Market unc.
Avg. time-series s.d.	0.0474	0.0803
Avg. cross-sectional s.d.	0.0342	0.0322
Frac. of var. explained		
by cross-sectional mean	0.60	0.86
Avg. pairwise corr.	0.82	0.92

The results here show that the finding of stability is not unique to the US: cross-sectional uncertainty has been similarly stable in other major developed economies. Second, there appears to be a strong international factor in cross-sectional uncertainty: uncertainty shocks have been global in nature over the last 18 years.

7 Conclusion

This paper reports a novel real-time index of cross-sectional implied volatility. A large literature studies the effects – both good and bad – of variation in the cross-sectional distribution of shocks that firms face. There is theoretical ambiguity about the effects of changes in cross-sectional uncertainty, but many policymakers take the view that uncertainty represents a hindrance to economic growth. It is thus an important empirical question not just what the time series of firm-level of uncertainty has looked like, but also whether shocks to cross-sectional uncertainty are in fact contractionary.

We develop a novel index of cross-sectional uncertainty with data extending back to 1980. The length of the sample is important – it is the data in the 1980's and early 1990's that emphasizes the extent to which the last two recessions have been anomalous. Prior to the late 1990's, there was little variation in cross-sectional uncertainty. Since then, there have been three episodes where it substantially grew, one a major economic expansion, with significant innovation, and the other two contractions. Studying raw correlations and forecasting regressions, we find that cross-sectional uncertainty is approximately acyclical overall and has little unconditional ability to forecast changes in future real activity. Overall, sometimes the data is consistent with models in which uncertainty shocks have causal negative effects on the economy, while in other periods it is consistent with models in which cross-sectional uncertainty is high following good shocks, for example due to a rise in innovation and creative destruction.

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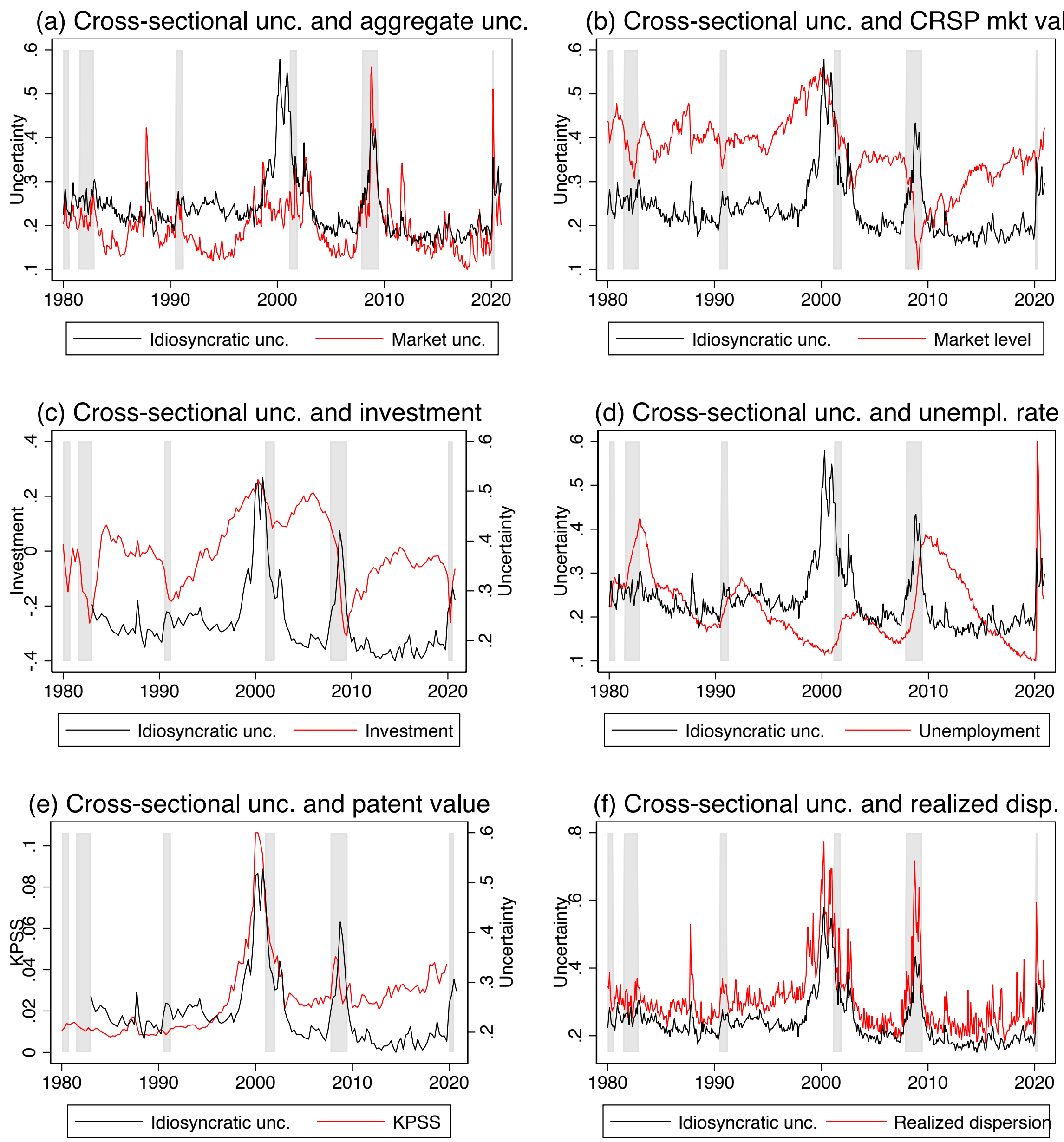
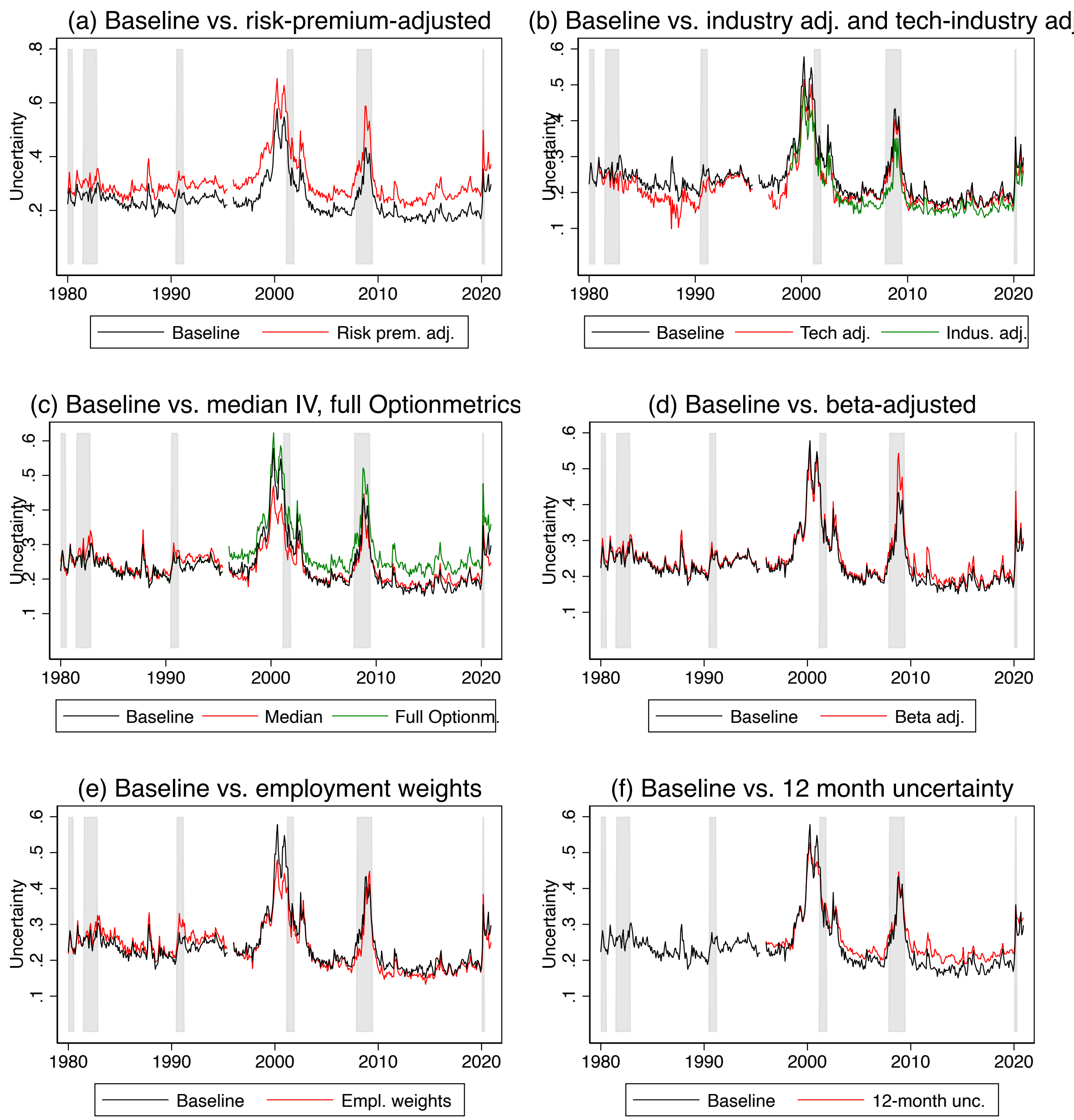
Figure 1: Time series of cross-sectional uncertainty



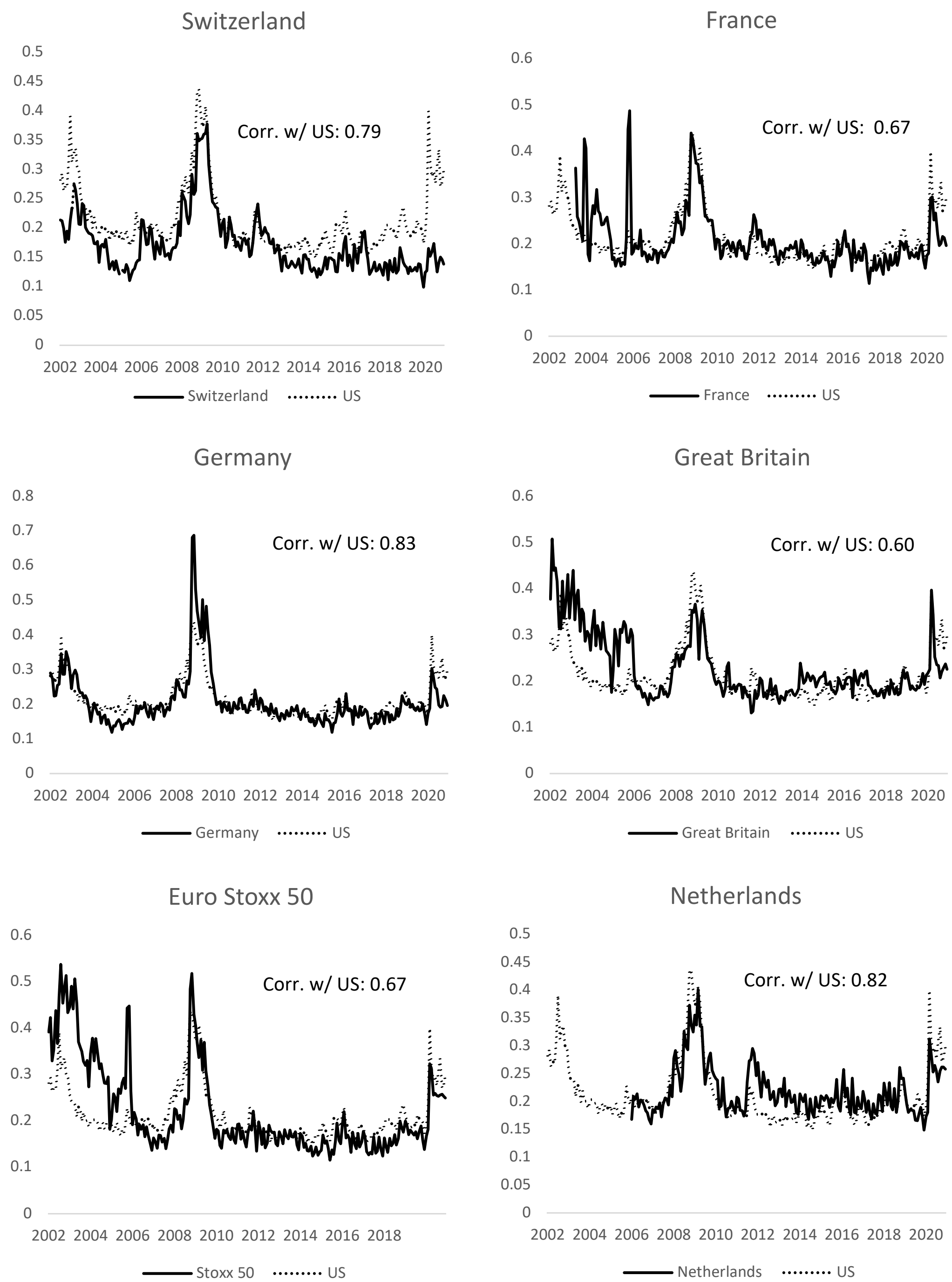
Note: Each panel plots cross-sectional uncertainty (darker line) together with another time series: aggregate uncertainty (panel (a)), stock market value (panel (b)), investment (panel (c)), unemployment (panel (d)), KPSS patent value/GDP (panel (e)), and realized dispersion (panel (f)). Options data before 1996 is from the Berkeley Options Dataset, for the period 1996/01-2020/12 is from Optionmetrics. The VIX (market uncertainty) is obtained from CME options. Shaded areas are NBER recessions.

Figure 2: Robustness



Note: The figure plots our baseline measure of cross-sectional uncertainty together with alternative measures built in different ways. Specifically: in panel (a), the alternative measure accounts for time-varying risk premia; in panel (b), it adjusts for industry and tech industry shocks; in panel (c), it uses the median of implied volatility across firms, instead of the weighted average by market cap; panel (c) also plots the measure obtained using all options in Optionmetrics instead of the largest 200; in panel (d), the alternative adjusts for betas; in panel (e), it uses employment weights instead of market cap; in panel (f), it uses 12-month instead of 30-day uncertainty.

Figure 3: Cross-sectional uncertainty across countries



Note: Cross-sectional uncertainty from option data in European markets (solid line) against the one for the US (dotted line). Data is from Optionmetrics until 2018, and from Bloomberg since 2019.

Table 1: Cyclicality of cross-sectional uncertainty

(a) Correlations

	Detrended IP	Detrended empl.	Unemployment rate	CBO output gap	NBER recession	Capacity utilitzation	GZ bond spread	IP growth	Employment growth	Change in unempl. rate	Change in output gap
Full sample	-0.05	0.05	-0.08	0.06	0.32	-0.02	0.43	-0.13	-0.08	0.08	-0.10
Pre-2020	-0.03	0.12	-0.11	0.09	0.32	0.01	0.43	-0.17	-0.25	0.23	-0.16
Pre-1/2008	0.09	0.15	-0.22	0.13	0.14	-0.12	0.52	-0.10	-0.12	0.09	-0.10
Post-1/2008	-0.36	-0.11	0.21	-0.29	0.78	-0.47	0.81	-0.27	-0.17	0.14	-0.15

(b) Forecasting realized dispersion

$$RD_t^x = b_0 + b_1 RD_{t-1}^x + b_2 \sigma_{\epsilon,t-1} + \eta_t$$

x:	Stock returns		IP growth IQR		Sales growth IQR	
$\sigma_{\epsilon,t-1}$	0.89	0.52	0.22	0.10	0.24	0.17
	[0.81,0.97]	[0.32,0.72]	[0.01,0.42]	[0.01,0.19]	[-0.05,0.52]	[-0.06,0.41]
RD_{t-1}^x		0.37		0.61		0.25
		[0.18,0.56]		[0.54,0.68]		[0.11,0.38]
$\#obs.$	449	449	449	449	149	149

(c) Uncertainty forecasting real activity, 1983–2019

$$y_t = b_0 + b_1 y_{t-1} + b_2 \sigma_{\epsilon,t-1} + b_3 RD_{t-1}^{ret} + b_4 \sigma_{mkt,t-1} + \eta_t$$

	ΔUnemployment			$\Delta log(Empl.)$			$\Delta log(IP)$		
$\sigma_{\epsilon,t-1}$	0.22	-0.00	0.09	-0.12	0.07	-0.02	-0.14	-0.12	-0.03
	[0.06,0.38]	[-0.24,0.24]	[-0.02,0.20]	[-0.21,-0.02]	[-0.08,0.23]	[-0.09,0.05]	[-0.28,-0.01]	[-0.32,0.08]	[-0.13,0.06]
RD_{t-1}^{ret}		0.23			-0.20			-0.02	
		[-0.03,0.49]			[-0.39,-0.01]			[-0.22,0.18]	
$\sigma_{mkt,t-1}$			0.25			-0.21			-0.21
			[0.07,0.44]			[-0.36,-0.06]			[-0.37,-0.04]
$\#obs.$	437	437	437	437	437	437	437	437	437

Note: Panel (a) reports correlations between cross-sectional uncertainty and various macroeconomic variables. Panel (b) reports the results of a regression of three different measures of realized cross-sectional dispersion on lagged cross-sectional uncertainty and lagged realized dispersion. The three measures are: realized cross-sectional dispersion of stock returns, the cross-sectional interquartile ranges of growth in industrial production (across sectors), and growth in sales (across Compustat firms). Panel (c) reports forecasting regressions of real activity, in the three sections, respectively: unemployment, change in employment, and change in industrial production. In each section, the first column uses lagged cross-sectional uncertainty as predictor, the second column adds lagged realized cross-sectional dispersion, and the last column adds instead market-wide uncertainty. In both sets of regressions, all variables are standardized to have unit variance. 90% confidence intervals calculated using Newey–West with 12 lags are reported in brackets.

Table 2: Comparing models to data moments

(a): Calibration moments

	Data	RUBC	CMR	Schaal	Di Tella	Gilchrist et al.
$SD[\sigma_{\varepsilon,t}]/E[\sigma_{\varepsilon,t}]$	0.30	0.71	0.58	0.14	0.20	0.09
	[0.17,0.38]					
$Corr[\sigma_{\varepsilon,t},\sigma_{\varepsilon,t-1}]$	0.91	0.91	0.98	0.94	0.71	0.90
	[0.81,1.02]					

(b) Correlations of growth rates with $\Delta\sigma_{\varepsilon,t}$

	Data	RUBC	CMR
GDP	0.035	-0.53	-0.34
	[-0.043, 0.113]		
Consumption	0.029	0.19	0.00
	[-0.050, 0.108]		
Investment	0.068	-0.50	-0.39
	[-0.005, 0.140]		
Hours worked	0.079	-0.72	-0.17
	[-0.004, 0.161]		

(c) Regressions

$$CMR:\Delta y_t = b_0 + b_1\Delta y_{t-1} + b_2\Delta\sigma_{\epsilon,t} + b_3\Delta^{IV}RD_t^{ret} + \eta_t$$

$$RUBC:\Delta y_t = b_0 + b_1\Delta y_{t-1} + b_2\Delta\sigma_{\epsilon,t} + b_3\Delta\sigma_{mkt,t} + \eta_t$$

y		Data	CMR		Data	RUBC
log GDP:	$\Delta\sigma_{\epsilon,t}$	0.011	-0.13	$\Delta\sigma_{\epsilon,t}$	0.005	-0.44
		[-0.000, 0.022]			[-0.001, 0.011]	
	$\Delta^{IV}RD_t^{ret}$	-0.015	0.21	$\Delta\sigma_{mkt,t}$	-0.010	-0.10
		[-0.031, 0.001]			[-0.021, -0.000]	
log Consumption:	$\Delta\sigma_{\epsilon,t}$	0.009	0.03	$\Delta\sigma_{\epsilon,t}$	0.006	0.10
		[-0.003, 0.020]			[-0.001, 0.013]	
	$\Delta^{IV}RD_t^{ret}$	-0.012	0.12	$\Delta\sigma_{mkt,t}$	-0.011	0.10
		[-0.027, -0.003]			[-0.022, -0.000]	
log Investment:	$\Delta\sigma_{\epsilon,t}$	0.035	-0.16	$\Delta\sigma_{\epsilon,t}$	0.008	-0.38
		[0.005, 0.065]			[-0.005, 0.021]	
	$\Delta^{IV}RD_t^{ret}$	-0.042	0.18	$\Delta\sigma_{mkt,t}$	-0.012	-0.13
		[-0.082, -0.002]			[-0.033, 0.010]	
log Hours:	$\Delta\sigma_{\epsilon,t}$	0.053	-0.12	$\Delta\sigma_{\epsilon,t}$	0.021	-0.59
		[0.016, 0.090]			[0.002, 0.039]	
	$\Delta^{IV}RD_t^{ret}$	-0.061	0.03	$\Delta\sigma_{mkt,t}$	-0.027	-0.14
		[-0.111, -0.011]			[-0.056, 0.002]	

Note: Panel (a) reports moments of the time-series of $\sigma_{\epsilon,t}$ in the data and in various papers. Panel (b) reports correlations of $\sigma_{\epsilon,t}$ with macroeconomic variables in the data and in the models. Panel (c) reports the results of regressions in the data and in the models. RUBC is Bloom et al. (2018); CMR is Christiano, Motto, and Rostagno (2016). For CMR $\sigma_{\epsilon,t}$ is the conditional cross-sectional standard deviation of stock returns, while in RUBC the two uncertainty processes are the conditional standard deviations of firm-specific and aggregate technology shocks. Δ denotes the first-difference operator, while $\Delta^{IV}RD_t^{ret}$ represents the innovation $RD_t^{ret}-\sigma_{\epsilon,t-1}$. All variables are at the quarterly frequency. In both sets of regressions, all variables are standardized to have unit variance. 90% confidence intervals calculated using Newey–West with 6 quarterly lags are reported in brackets.

A.1 Constructing implied volatility

For the Optionmetrics sample, we obtain at-the-money implied volatilities as the delta=50 IVs with maturity of 30 days from the Optionmetrics surface file.

In the period since 2009, there has been an increase in the seasonality of implied volatility around earnings announcement dates. For the period 2010–2020, we therefore estimate a nonlinear regression for average firm-level implied volatility that fits a sine curve to the data with precisely four cycles per year. That sine curve is then removed to yield the seasonally adjusted series.

For the BODB, the steps are as follows:

1. We calculate closing bid and ask prices for each option as the average of the final value and any other values recorded in the last 15 minutes of trading.

2. For each date/maturity/ticker combination, we take the strike immediately above and below the underlying price, as long as it is within 20 percent of the underlying.

3. Option prices are calculated as the midpoint between the bid and ask.

4. We drop all options with maturity less than 7 days or where the quoted price is less than the intrinsic value.

5. The BODB reports a spot price. We replace the spot price with the value implied by put-call parity with a dividend of zero if the put-call parity implied price differs from the reported spot by more than 20 percent (this is to eliminate some clear data errors).

6. Implied volatilities are constructed using the Black–Scholes formula for European options ignoring dividends. For the one-month maturity, early exercise has generally very small effects on prices. We experimented by using the same method on data from Optionmetrics and comparing it to the implied volatilities that they report (which use a model for dividends and also account for early exercise) and we found the differences were quantitatively small.

7. We interpolate between maturities – and extrapolate where necessary – to get 30-day implied volatilities. Firm-level implied volatilities are set to have a maximum of 200 percent annualized and a minimum of zero (the interpolated values are Winsorized).

8. The implied volatilities are then collapsed across firms weighting by market capitalization. We matched the tickers in the BODB to CRSP permco numbers to get market capitalization. In the large majority of cases, the BODB tickers are the same as the stock exchange tickers (they differ most for NASDAQ listings; the BODB manual, available online or on request from us, discusses this issue). The remainder are matched by hand where possible.

10. Between 2010 and 2019, there appears to be some seasonality in firm-level implied volatility, likely due to earnings announcements. We therefore remove a sinusoidal component

from the firm-level implied volatilities with a period of one quarter – this smooths out the monthly data but has no impact on quarterly data.

A.2 How much of the variation is common?

For most of the analysis, we follow the literature in studying the common component in cross-sectional uncertainty. It is worth asking, though, how much of the variation in firm-level uncertainty is driven by that common component. To do so, we use the law of total variance,

$$\underbrace{\operatorname{var}\left(x_{i,t}\right)}_{\text{Total variance}} = \underbrace{E\left[\operatorname*{var}_{t}\left(x_{i,t}\right)\right]}_{\text{Average cross-sectional variance}} + \underbrace{\operatorname{var}\left[E_t\left(x_{i,t}\right)\right]}_{\text{Time-series variance of the average}} \tag{A.1}$$

where var_t and E_t refer to the cross-sectional variance and average on date t. The first term represents the residual variance after accounting for the cross-sectional average in each period, while the second term is the variance coming from that average. So the ratio of $\operatorname{var}\left[E_t\left(x_{i,t}\right)\right]$ to the total variance represents the fraction of the total variance explained by the cross-sectional mean in each period.

The variance decomposition identity (A.1) also holds with weights, so we weight by market capitalization as above (normalizing the sum of market capitalization to 1 on each date to give them equal weight overall). For $x_{i,t}$, we use total firm implied volatility measured here as

$$\sigma^2_{R,\varepsilon,i,t} \equiv \sigma^2_{i,t} - \beta^2_{i,R,t}\sigma^2_{mkt,t} \tag{A.2}$$

where $\sigma^2_{R,\varepsilon,i,t}$ is a rolling beta estimated using the previous 12 months of daily data. When we are just calculating the average of cross-sectional uncertainty across firms, the errors from setting $\beta_i \approx 1$ somewhat cancel out across firms. Here, though, those errors will affect the variance decomposition, so it is important to also examine what happens when we actually estimate β_i.

Variance decomposition for uncertainty measures

	Fraction from common component	
	Firm level	Sector level
Total firm uncertainty ($\sigma^2_{i,t}$)	0.50	0.75
Firm-specific uncertainty ($\sigma^2_{R,\varepsilon,i,t}$)	0.40	0.70

Depending on the measure, between 40 and 50 percent of the total variation in uncertainty is due to a common component (measured as the cross-sectional average), which is quantitatively consistent with the results in Herskovic et al. (2016). The fraction explained by a common component is greater for total firm uncertainty, which is natural since that includes market uncertainty, which affects all firms. The second column of the table above reports similar results for measures of uncertainty averaged within two-digit sectors (i.e. $\sum_{i \in S} w_{i,t} \sigma^2_{R,\varepsilon,i,t}$, where S represents the set of firms in some sector). These results therefore measure the extent to which cross-sectional uncertainty is similar across different sectors. In this case, the fraction of the variation explained by the time-series component is 1.5 times larger than in the firm-level case.

Overall, then, a surprisingly large amount of the total variation in cross-sectional or idiosyncratic risk is driven by a common component that hits all parts of the economy, which motivates us (and previous authors) to study a single common factor.

A.3 Accounting for risk premia

In this appendix, we use $\sigma^2_{\varepsilon,t}$ to denote the true firm-specific uncertainty, while $IV^2_{\varepsilon,t}$ denotes the option-implied measure that the main results focus on. We can define the cross-sectional variance risk premium as

$$XVRP_t = \sigma^2_{\varepsilon,t} - IV^2_{\varepsilon,t} \tag{A.3}$$

Now suppose that risk premia are determined by some set of variables $\{x_{j,t}\}$, with

$$XVRP_t = a_0 + a_{IV} IV^2_{\varepsilon,t} + \sum_j a_j x_{j,t} \tag{A.4}$$

Combining equations (A.3) and (A.4) yields

$$\sigma^2_{\varepsilon,t} = a_0 + (a_{IV} + 1) IV^2_{\varepsilon,t} + \sum_j a_j x_{j,t} \tag{A.5}$$

That is, when risk premia have the linear specification of (A.4), the true conditional variance of cross-sectional shocks is a linear function of the option-implied variance and the other state variables. As mentioned in the text, in the case where risk premia are constant or proportional to $IV_{\varepsilon,t}$, $\sigma^2_{\varepsilon,t}$ is perfectly correlated with $IV^2_{\varepsilon,t}$, so that the cyclicality of $\sigma^2_{\varepsilon,t}$ is identical to that of $IV^2_{\varepsilon,t}$ (in terms of correlations, not magnitudes).

Now since $\sigma^2_{\varepsilon,t}$ is, by assumption, the true conditional variance, we have:

$$\sum_i w_{i,t} \varepsilon^2_{i,t+1} = E_t \left[\sum_i w_{i,t} \varepsilon^2_{i,t+1} \right] + \eta_{t+1} = \sigma^2_{\varepsilon,t} + \eta_{t+1}$$

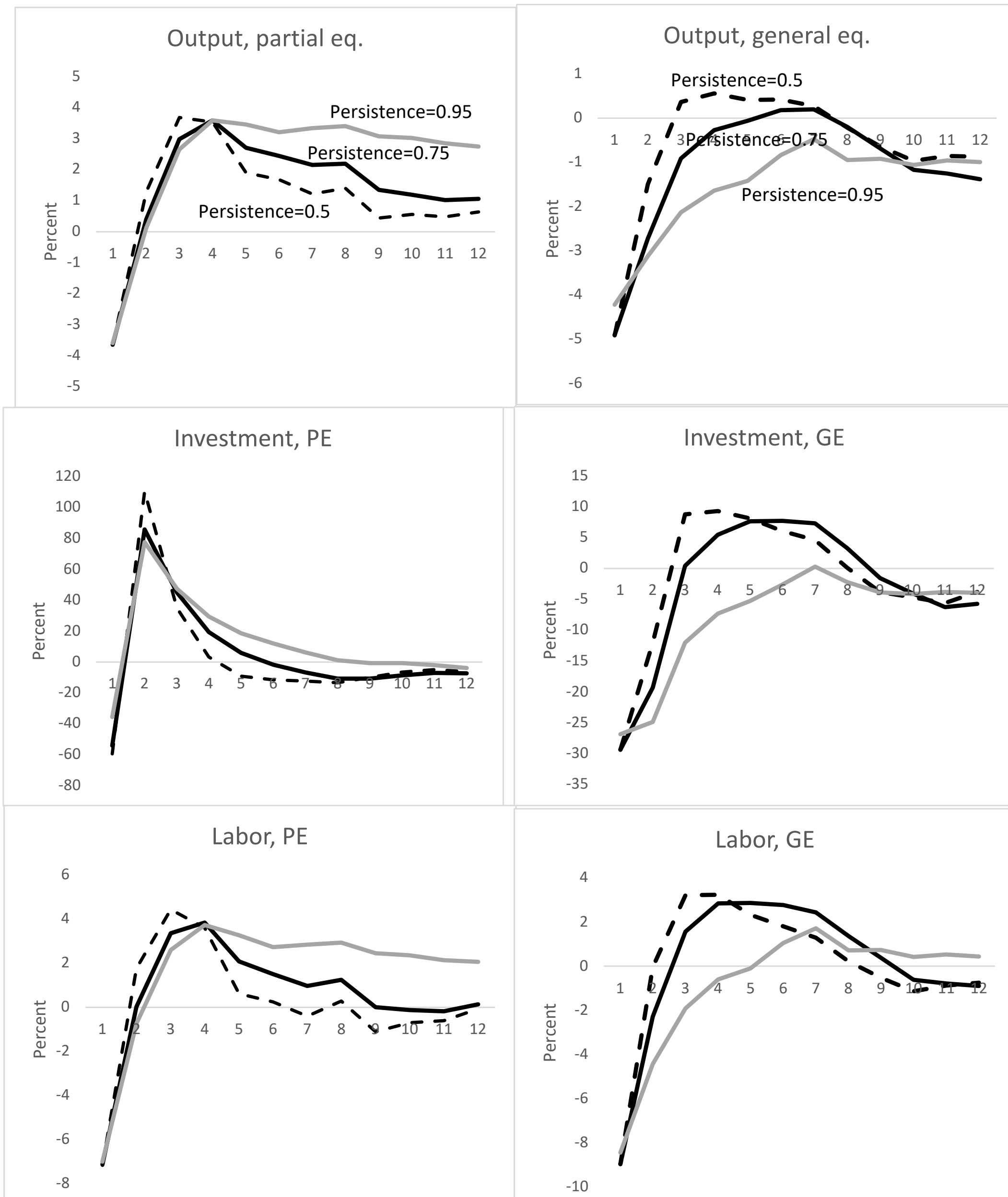
where η_{t+1} is a residual that is orthogonal to any date-t or earlier variables, and hence

$$\sum_i w_{i,t}\varepsilon_{i,t+1}^2 = a_0 + (a_{IV}+1)\, IV_{\varepsilon,t}^2 + \sum_j a_j x_{j,t} + \eta_{t+1} \tag{A.6}$$

That shows that $\sigma_{\varepsilon,t}^2$ can be recovered as the fitted value from a regression of $\sum_i \omega_{i,t}\varepsilon_{i,t+1}^2$ on $IV_{\varepsilon,t}^2$ and the other state variables, $\{x_{j,t}\}$ (up to estimation error in the coefficients $\{a_j\}$).

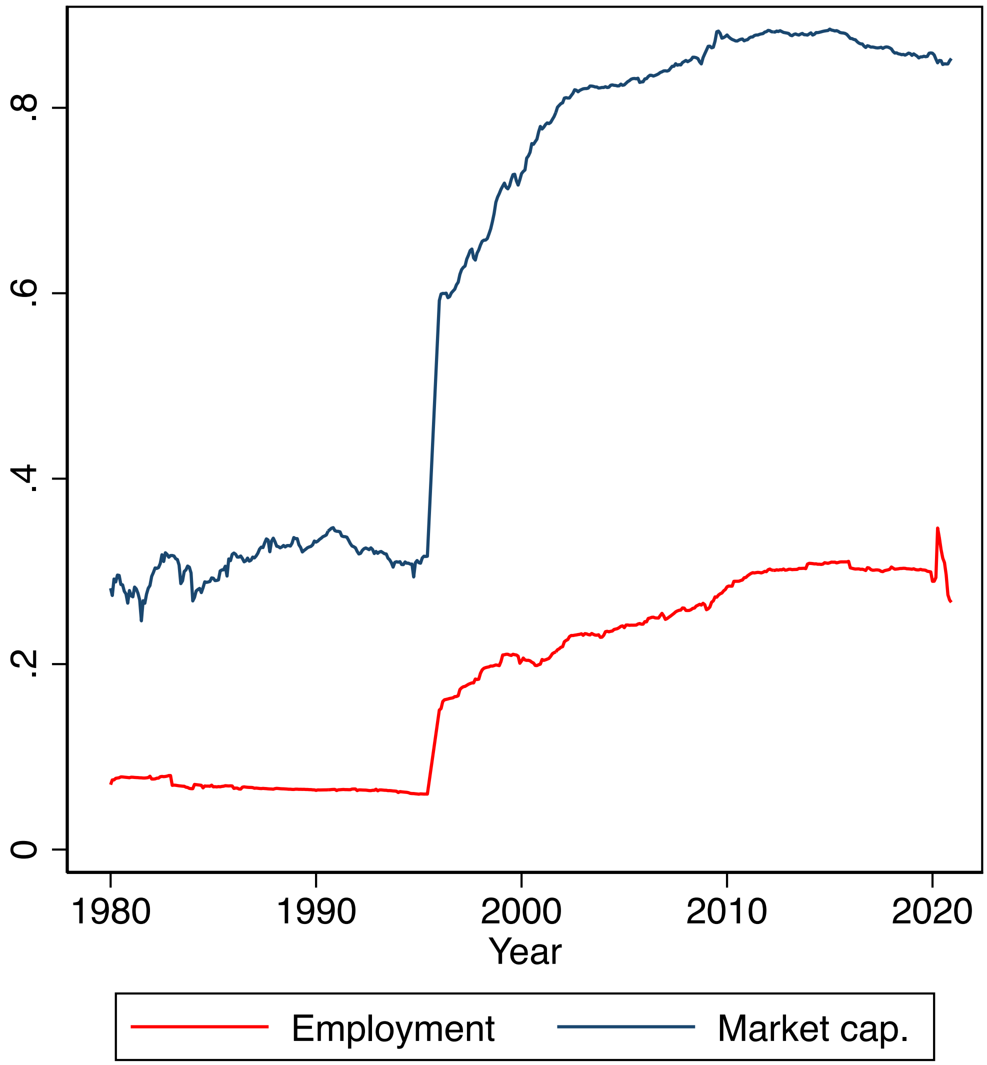
The fact that we find that the behavior of $IV_{\varepsilon,t}$ and the fitted values from the regression above is very similar is due to the $\{a_j\}$ coefficients, other than a_0 and a_{IV}, being quantitatively small. a_0 is what induces the level difference between the two series and is evidence of a constant risk premium. The net result is that the fitted value from the regression is over 95 percent correlated with $IV_{\varepsilon,t}^2$.

Figure A.1: IRFs in the RUBC model



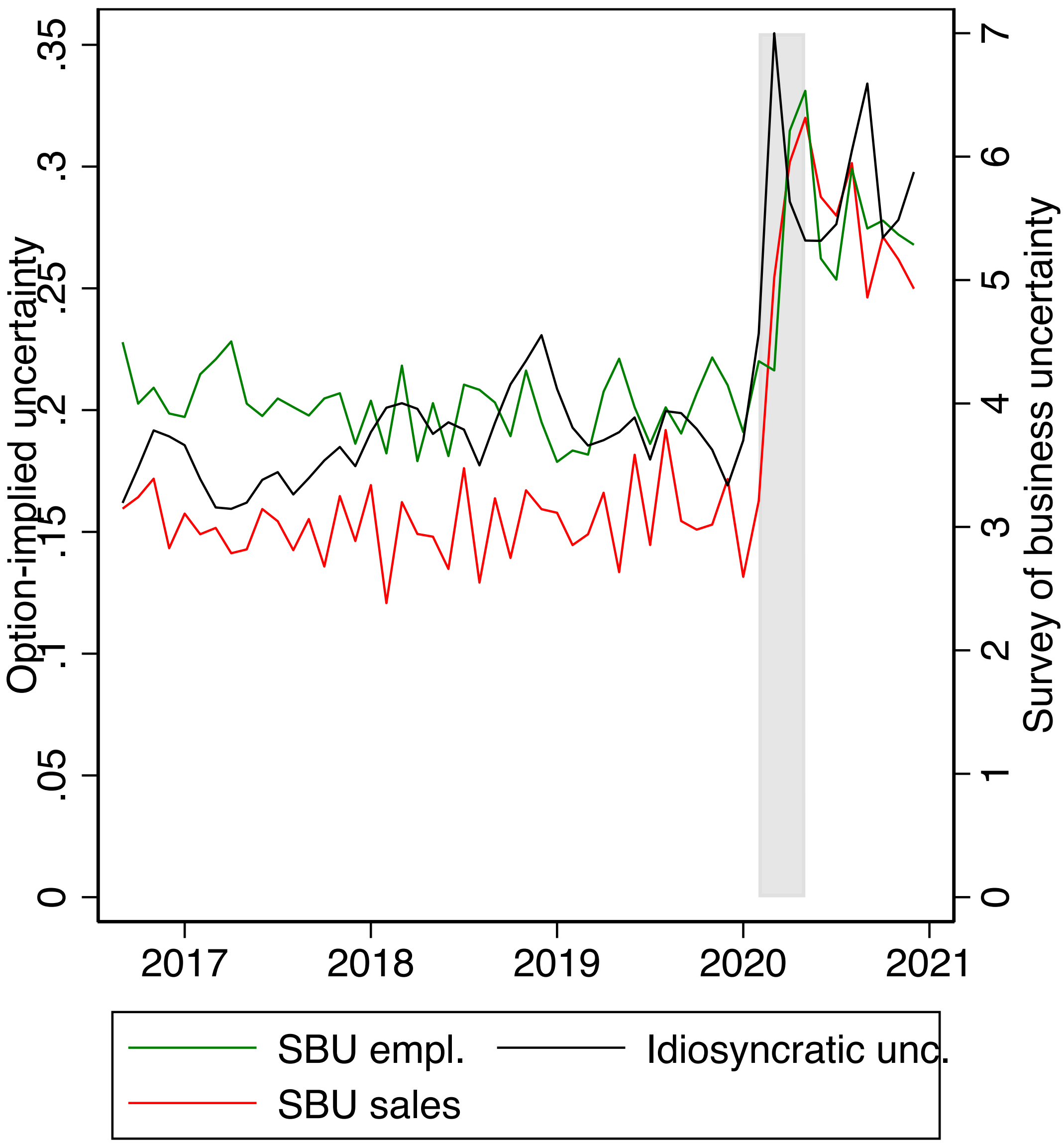
Note: The figure reports the partial-equilibrium (left side) and general equilibrium (right side) IRFs of output, investment, and labor to uncertainty shocks of different persistence of the volatility shock, in the RUBC model. The period length is one quarter. IRFs are calculated as the difference in the mean conditional on a transition from low to high uncertainty in period 1. Persistence is the probability of the Markov chain remaining in the high state.

Figure A.2: Fractions of market capitalization and employment covered by the options data



Note: The figure reports the ratio of total market capitalization for the firms for which we observe options data to the total market capitalization (dark line), and the fraction of aggregate employment covered (lighter line). Data before 1996 is from the Berkeley Options Dataset, and data after 1996 is from Optionmetrics.

Figure A.3: SBU



Note: The figure plots our baseline measure of cross-sectional uncertainty together with the sales and employment uncertainty from the Federal Reserve Bank of Atlanta's Survey of Business Uncertainty (SBU) over the period 2016-2020.

Table A.1: Cyclicality of cross-sectional regressions and forecasting results

(a) Correlations

	HP IP, 129,600	HP IP, 518,400	Exp MA IP	HP Emp., 129,600	HP Emp, 518,400	Exp MA Emp.
Full sample	-0.05	-0.02	-0.18	0.05	0.07	-0.16
Pre-1/2008	0.08	0.12	-0.05	0.15	0.16	-0.07
Post-1/2008	-0.36	-0.39	-0.71	-0.11	-0.13	-0.55

Note: Replicates table 1a, but with alternative detrending for IP and employment. HP IP, 129,600 is HP-filtered IP with a smoothing parameter of 129,600, and the other columns are similar. "Exp MA" is a case where we detrend the growth rates using an exponentially weighted moving average filter, with a decay rate of 10 percent per month, and then cumulate the detrended growth rates to recover the level.

Table A.2: Forecasting cross-sectional standard deviations

x:	IP growth SD			Sales growth SD		
$\sigma_{\epsilon,t-1}$	0.18	0.09	-0.24	0.21	0.14	-0.17
	[-0.01,0.38]	[-0.02,0.20]	[-0.77,0.29]	[-0.05,0.48]	[-0.05,0.32]	[-0.66,0.31]
RD^x_{t-1}		0.63			0.37	
		[0.57,0.70]			[0.27,0.48]	
$RD^{ret_{t-1}}$			0.47			0.41
			[-0.19,1.13]			[0.06,0.77]
$\#obs.$	449	449	449	149	149	149

Note: Replicates table 1b, but replacing the interquartile ranges with cross-sectional standard deviations.