Skewness and time-varying second moments in a nonlinear production network: theory and evidence

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Abstract

This paper studies asymmetry in econoimc activity over the business cycle. It develops a tractable multisector model of the economy in which complementarity across inputs causes aggregate activity to be skewed left. We then examine six further implications of the model regarding the time-series skewness of activity at the sector level, cyclicality of dispersion and skewness across sectors, and the conditional covariances of sector growth rates and find support for all six in the data. Other prominent models of asymmetry are not able to simultaneously match the range of empirical facts that the network production model can.

1 Introduction

Background

A defining feature of the business cycle is the existence of recessions as distinct episodes. Rather than simply experiencing symmetric random fluctuations around a trend, output, unemployment, and other measures of the state of the economy display sharp declines and relatively smooth expansions: levels and growth in real activity, and also stock returns, are skewed left.[1] At the same time, many measures of volatility, in the aggregate and the cross-section, and for real quantities, prices, and asset values, are countercyclical. As a mathematical matter there is a mechanical link between skewness and countercyclical

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[1] See Sichel (1993), McKay and Reis (2008), Morley and Piger (2012), Berger, Dew-Becker, and Giglio (2019), and Dupraz, Nakamura, and Steinsson (2019) among others.

volatility – high volatility in bad times leads to a long left tail of outcomes – but few models capture that feature of the economy.

This paper takes that asymmetry as its starting point. Following Baqaee and Farhi (2019), we develop tractable a multisector production model in which inputs to production are complements. The model then yields a number of testable predictions that we study empirically.

Model

We develop a model building on the tradition of multisector production models starting with Long and Plosser (1983). In almost all past work, the elasticity of substitution across inputs in production has been assumed to be 1, so that the model is log-linear and can be solved by hand. Baqaee and Farhi (2019), however, argue, based on local approximations and numerical simulations, that allowing non-unitary elasticities is important for understanding a range of facts about the economy, including business cycle asymmetry.

To build intuition, consider an extreme case in which aggregate output is Leontief in a range of inputs (consistent with the estimates of Barrot and Sauvagnat (2016), Atalay (2017), and Carvalho et al. (2016)), and that production of the inputs depends only on sector-specific productivity shocks. Then the distribution of output is the involves the minimum of the technology shocks, which is in general skewed left. With network effects, that minimum – the productivity of the worst-performing sector – becomes a common component that affects all sectors.

We study a relatively stylized production network that has the advantage that it is analytically tractable, yielding at least six specific empirical predictions, in addition to the original goal of generating left skewed aggregate activity. Formally, the solution to the model yields a simple linear factor model for sector output, employment, and payments to capital, and the common factor is skewed.

The model's key predictions are as follows:

1. Sector activity is skewed left, but by a smaller degree than aggregate activity
2. The sector-specific component of activity is unskewed
3. The cross-sectional variance of sector activity is countercyclical
4. The cross-sectional skewness of sector activity is procyclical

5, 6, 7. When a sector receives a negative shock, it subsequently covaries more strongly with other sectors (5) and with aggregate activity (6) and becomes more volatile itself (7)

In the model, fundamental shocks to sectors are symmetrical, generating the second fact. Total sector output depends both on the sector shock and aggregate output. That sum is always less skewed than aggregate output, generating fact 1.

Countercyclical cross-sectional variance and procyclical skewness are generated simply because the minimum of the sector-level shocks tends to be lower in periods when the sample standard deviation happens to be higher or the skewness is lower. The model is thus able to generate countercyclical volatility – which is sometimes taken as evidence for countercyclical uncertainty – even though all shocks are homoskedastic, calling into question empirical analyses of uncertainty shocks based on cross-sectional volatilities (e.g. see the discussion in Bloom (2014)). It also builds on the recent work os Salgado et al. (2019) in analyzing cross-sectional skewness.

The final two facts are included because they directly address the key mechanism in the model. In a model with complementarities, when the supply of one input shrinks, that input becomes more important, in the sense that output becomes relatively more sensitive to it and less sensitive to other inputs. Because the sectors that shrink become more relevant, they also covary more strongly with any other sector that buys their output (including final production).

A single simple idea, then, that production features complementarities, generates a number of testable predictions that can be used to compare the model both to the data and to other models.

Empirical results

We begin by providing evidence on time-series asymmetry in various measures of aggregate activity. The evidence is statistically strongest for industrial production and stock returns, but the point estimates for skewness are negative for all series we look at, including employment, GDP, consumption, and investment, both in levels and growth rates.

The next step is to test the six predictions of the model. All six find support in the data. We focus on monthly data, since it gives us the most power in measuring asymmetry and conditional moments, so the main data sources we study are industrial production, employment, and stock returns.

For all three sets of data, we find negative skewness at both the aggregate and sector level. Consistent with the model, however, the magnitude of the skewness increases with the level of aggregation – aggregate activity has skewness that is at least twice as negative as sector-level activity. Furthermore, when we examine residuals from regressions of sector on aggregate activity, they are symmetrically distributed. In other words, all the skewness observed at the sector level can be explained purely by exposure to an aggregate factor. So skewness seems to be an aggregate phenomenon, rather than being due to, for example, skewed micro shocks.

The third and fourth facts, on the cyclicality of cross-sectional moments, have been studied previously. We confirm that they also hold in the data we study, using multiple different measurement methods.

Finally, the two predictions for time-varying covariances are entirely novel to this paper. We therefore test them using a number of different methods to ensure robustness. For employment and industrial production, following a positive statistical innovation in output in a given sector and month (which, in the model, identifies a sector productivity shock), that sector covaries less strongly with other sectors and with aggregate activity for the next three to 12 months. Those covariances are measured relatively poorly in monthly data, though. Stock returns allow us to use high-frequency data to measure covariances, and we obtain similar results. Finally, we use the NBER-CES manufacturing database to measure shocks to total factor productivity in each sector, and again find support for the model's predictions for how covariances respond to shocks.

Alternatives

Since our starting point is business cycle asymmetry, our last question is whether other models that generate such asymmetry can also match the six additional predictions that this paper generates and tests. We examine three basic specifications:

a. Skewed aggregate shocks (or perhaps skewness in a universal input, such as finance, as in Brunnermeier and Sannikov (2014));[2]

b. Stochastic volatility and skewness in aggregate and idiosyncratic shocks (Bloom (2009));

c. Concave responses to shocks (Ilut, Kehrig, and Schneider (2018)).

Each of the models, under suitable restrictions, is able to generate a subset of the facts that we document, but they all fail on at least one. The comparison with the alternative models also helps illustrate precisely why the network model is able to fit the data. In brief, facts 1 and 2 on time series skewness point to the existence of a skewed common component in output – that is, skewness arises at the aggregate rather than micro level – while facts 3 and 4 on the cross-section imply that the common component is endogenous to sector-level shocks. Explicit aggregation is therefore central because it creates that endogeneity. There always exists some sufficiently rich purely statistical specification that can match any set of empirical facts, and the facts could also likely be matched by some combination of the alternative models we study, but the aggregative model is notable for fitting the facts parsimoniously, relying essentially just on complementarity.

Implications

The results in this paper have important implications for how to think about volatility

[2] The literature on rare disasters, e.g. Barro (2006), Gourio (2012, 2013), and Wachter (2013)

and skewness in the both the real economy and financial markets.[3] A large literature studies stochastic volatility (and, more recently, stochastic skewness), often implicitly assuming that because the dispersion in observed distributions changes over time, that means that there are "uncertainty shocks" (e.g. Bloom et al. (2018)). The results here show that time-variation in sample moments – and their cyclicality – need not have anything to do with uncertainty. Rather, increases in the dispersion of realized shocks may simply be associated with declines in output because of strong concavity in the production function.

The model and empirical results also demonstrate that the economy in an important sense does not have a single fixed network structure – in terms of the covariance of output across sectors – and in fact that the variation is important in causing asymmetries in outcomes. Baqaee and Farhi (2019) study this point extensively, and our results provide further empirical support. They show that complementarity can generate aggregate skewness and variation in the centrality of sectors. The six empirical facts – on sector level output, cross-sectional volatility, and conditional covariances – are all steps beyond their work, and necessary for distinguishing other models.

Activity is skewed left and stochastically volatile in the model precisely because a single sector – or set of sectors – will occasionally receive a negative shock and become (approximately) a limiting factor in production. It is exactly that change in the network – that the negatively shocked industry becomes central – that creates a recession, negative skewness, and volatility. So while it is common for studies of economic networks to use snapshots of the inter-sector trade structure from the input-output tables, such an analysis can miss important variation in linkages, and this paper shows that the variation, which produces the time-varying second moments – is empirically relevant.

Finally, and perhaps most importantly, the analysis helps understand why recessions exist as discrete events. Hypotheses like skewness in aggregate shocks or concavity in firm responses to shocks have been proposed to match the steepness and deepness of recessions, but they turn out to fail on other dimensions. The necessary concavity arises naturally when there is complementarity in production.

Related work

The paper is related to a number of active literatures. First, as described above, is work on time-variation in time-series and cross-sectional moments of output. Second is an emerging

[3]For work on unconditional skewness, see Sichel (1993), McKay and Reis (2008), Morley and Piger (2012), and Berger, Dew-Becker, and Giglio (2019). For recent work on time-varying volatility in the real economy, see Justiniano and Primiceri (2008), Clark and Ravazzolo (2015), and Schorfheide, Song, and Yaron (2018). work on time-varying cross-sectional moments includes Guvenen, Ozkan, and Song (2014), Salgado et al. (2018), and Dew-Becker and Giglio (2019).

literature on production networks with complementarities in production. Atalay (2017) and Atalay, Drautzburg, and Wang (2018) estimate the elasticity of substitution using data on sector production and input-output linkages, finding extremely strong complementarities (up to the point of a Leontief production function). Barrot and Sauvagnat (2016) find similar results in firm-level responses to natural disasters. Baqaee and Farhi (2019) theoretically study models with complementarities and show in a calibration that they can generate a realistic level of skewness at the aggregate level, in addition to studying how covariances and the network structure change over time in general production frameworks. Carvalho et al. (2016) study the economy of Japan following the 2011 earthquake in a network model with non-unitary elasticities of substitution. Our contribution relative to this literature is in showing that an aggregative model is able to match the four stylized facts on skewness and to compare its performance to other leading potential explanations.

The remainder of the paper is organized as follows. Section 5 documents the three empirical facts. Section 3 presents a simple and stylized aggregative model with complementarities and formally shows that it can match the three empirical facts. In section 7 we examine the predictions of three other models of economic skewness. Section 6 examines a model with complementarities in a more quantitatively realistic calibration, and section 8 concludes.

2 Skewness in aggregate activity

The fact that aggregate output and employment – in both levels and growth rates – along with stock returns, are skewed left has been established in previous work.[4] This section provides a brief overview of some evidence on asymmetry in aggregate activity.

Table 1 reports skewness coefficients for levels and growth rates of six measures of aggregate activity: GDP, consumption, investment, industrial production, employment, and stock returns. p-values for a two-sided test against the null of zero skewness from a block bootstrap are reported in brackets. Across the six series, in both levels and growth rates, the skewness coefficients are negative in all cases. Statistically, the evidence is somewhat weak, which is a consequence of the facts that higher-order moments are typically difficult to estimate well and that our sample is only 47 years long. The magnitude of the point

[4] Berger, Dew-Becker, and Giglio (2019), show that growth rates of employment, capacity utilization, industrial production, GDP, durable and non-durable consumption, and residential and nonresidential investment are all skewed left. Furthermore, returns on the S&P 500 are skewed left, as is their option-implied distribution. Morley and Piger (2012) provide a much more thorough analysis of asymmetry in the output gap – that is, on skewness in levels, rather than growth rates – and finding similar results – while Sichel (1993) provides an earlier analysis distinguishing asymmetry in levels from growth rates. See also references therein for the literature on business cycle asymmetry.

estimates is relatively large, averaging -0.77, ranging from -0.36 to -1.22.

The remaining columns report results for a nonparametric measure of asymmetry. Denote the mean and standard deviation of some variable x by μ_x and σ_x, respectively, and the empirical cumulative distribution function as $\hat{F}_x(z)$. The table reports $\hat{F}_x(\mu_x - k\sigma_x) / \left(1 - \hat{F}_x(\mu_x + k\sigma_x)\right)$ for various values of k. That ratio measures the relative probability that x is k standard deviations below its mean compared to the probability it is k standard deviations above its mean. If the left tail is asymptotically heavier than the right, in the sense that the probability ratio diverges to ∞, then choosing a large value of k will produce a larger ratio. At the same time, though, when k is larger the probability ratio is calculated based on fewer observations. So the ability to reject the null that the ratio is equal to 1 will, heuristically, peak for some finite k (for $k = 10$, for example, the sample CDFs will both be equal to zero and the ratio undefined – we have no 10-standard deviation events in our sample).

The cutoff k ranges in the table between 1 and 3, with power appearing to peak around values of 1.5 and 2. In all cases, as k rises, the relative probability of negative deviation rises. In other words, large negative deviations in both levels and growth rates of the six variables studied in table 1 have been far more common than large positive deviations. At $k = 1.5$ and $k = 2$, a number of the ratios are statistically significantly larger than 1 at conventional levels, but that statistical significance is not uniform across all estimates. Compared to the results for skewness, the tail probability ratios are relatively more weakly measured statistically. So while the tail measures are attractive for being simple and nonparametric, they face the usual tradeoff of also having somewhat lower power.

Overall, then, table 1 shows that in the empirical sample we study, major measures of aggregate activity are consistently skewed left according to a range of measures. While the results are individually only marginally statistically significant, they overall tell a consistent story of a long left tail, and we take that left skewness as the basic starting point for the remainder of the analysis.

3 Nonlinear network model

This section presents a model that can generate the negative skewness in aggregate activity presented in the previous section. It is a production network, building on the work of Long and Plosser (1983), so it will yield predictions both about aggregate and sector-level output. It allows for nonlinearity in production – through an elasticity of substitution that can differ from 1 – in order to generate nonlinearities in the distribution of activity. Conditional on featuring a nonlinear production network, we set the model up to be as simple as possible, which will allow us to obtain transparent analytic solutions and formally derive a number of testable predictions. Section 6 examines results from a numerical solution of a richer and more realistic specification to check whether the predictions derived here are robust.

3.1 Skewness in generic network models

We begin with a very general production specification. In each sector, indexed by i, output is produced according to the function

$$Y_{i,t} = \exp\left(\varepsilon_{i,t}\right)\left[\left(1-\alpha_i\right)N_{i,t}^{\beta_i} + \alpha_i\left(\sum_j a_{i,j}x_{i,j,t}^{\gamma_i}\right)^{\beta_i/\gamma_i}\right]^{1/\beta_i} \tag{1}$$

where $\varepsilon_{i,t}$ is sector i's productivity on date t, N_i is sector i's use of labor, $x_{i,j}$ is sector i's use of material input j, γ_i and β_i respectively determine elasticities across material inputs and between inputs and labor, and $a_{i,j}$ and α_i determine the relative importance of inputs. This specification can accommodate a wide range of possible production specifications, allowing for elasticities of substitution and input mixes that vary across sectors.

Final consumption is

$$C_t = \left(\sum_j a_{C,j}x_{C,j,t}^{\gamma_C}\right)^{1/\gamma_C} \tag{2}$$

where $x_{C,j,t}$ is consumption of good j. There is no investment and the economy is closed, so gross domestic product is equal to C_t.

The resource constraint for each sector is

$$\sum_i x_{i,j,t} + x_{C,j,t} = Y_{j,t} \tag{3}$$

Throughout the analysis, we assume that the $\varepsilon_{i,t}$ are distributed symmetrically around zero, both conditionally and unconditionally. Any skewness in log output will then arise endogenously through the production and aggregation process.

Proposition 1 *Subject to technical restrictions discussed in the appendix, if $\gamma_i \leq 0$ for all i, with the inequality being strict for at least two values of i, $\log C_t$ is unconditionally negatively skewed in the sense that*

$$\lim_{\bar{c}\to-\infty}\frac{\Pr\left[\log C_t < \bar{c}\right]}{\Pr\left[\log C_t > -\bar{c}\right]} = \infty \tag{4}$$

If the signs of the γ_i are reversed, then the limit is equal to zero and output is positively skewed.

Proposition 1 shows that for general production networks, when inputs are complements, the probability that output has a large negative deviation is substantially larger than the probability that it has a large deviation.[5] The intuition for the result is simple: CES aggregators with elasticities less than 1 approximate a minimum function, and they are limited by their smallest argument. A low realization of productivity in one sector is thus sufficient to limit aggregate output. So low output can occur if any shock is low, while high output requires all shocks to be high.

Proposition 1 refers to levels, but there are cases in which results can also be obtained in growth rates. Appendix C shows that in a continuous-time limit of the model, growth rates are also skewed left as long as the shocks to productivity have fat tails (jumps). In the absence of jumps, the local linearity of the production function causes the distribution of output growth to be symmetrical. We provide evidence below that sector-level shocks are in fact fat-tailed, and Atalay, Drautzburg, and Wang (2018) provide a more thorough analysis.

The remainder of the paper will analyze a special case of the model that is more tractable, but proposition 1 shows that the negative skewness that represents the initial motivation for the model is a highly robust feature.

3.2 Analytically tractable case

The general version of the model cannot be solved by hand, so we consider a special case. We set $\beta_i = 1$, so that production is Cobb–Douglas in labor and material inputs. We assume that the elasticities γ_i and γ_C are identical across sectors, equal to some γ. Similarly, $\alpha_i = \alpha$, so that labor's share of expenditures is constant across sectors. Finally, $a_{i,j} = a_{C,j} = a_j$, ensuring that all sectors use the same mix of inputs. While these assumptions are clearly counterfactual, they yield tractability.

There are at least three ways labor can be modeled: the aggregate supply can be fixed and it can adjust endogenously across sectors (e.g. Long and Plosser (1983)); it can be fixed within each sector (Baqaee and Farhi (2019)); or the real wage can be fixed and the quantity allowed to vary freely (e.g. Blanchard and Gali (2010)). We focus on the case where $N_{i,t}$ is fixed, as in Baqaee and Farhi (2019), (and we simply normalize it to 1), but we also examine results in the other two cases, discussed in section 4.4. The benchmark model thus involves the following three equations

[5] Baqaee and Farhi (2019) analyze a similar model using a second-order approximation. They provide expressions for skewness and discuss the case in which there are only shocks in a single industry. Unfortunately the general expression for skewness in that approximation is nearly a page long and involves five moments of the shock distribution, making it difficult to analze directly.

$$Y_{i,t} = \exp\left(\varepsilon_{i,t}\right) N_{i,t}^{1-\alpha} \left(\sum_j a_j x_{i,j,t}^{\gamma}\right)^{\alpha/\gamma}$$

$$C_t = \left(\sum_j a_j x_{C,j,t}^{\gamma}\right)^{1/\gamma}, \quad \sum_i x_{i,j,t} + x_{C,j,t} = Y_{j,t}$$

3.3 Solution

Since the model is static, in the sense that decisions made on date t have no impact on those on any other date, it can be solved date-by-date. Where there is no risk of confusion, we drop time subscripts from the analysis for the sake of concision.

The appendix shows that the model's solution for sector and aggregate output is, up to constant factors,

$$Y_i \propto \left(n^{-1}\sum_j a_j^{\frac{1}{1-\alpha\gamma}} \exp\left(\varepsilon_j\right)^{\frac{\gamma}{1-\alpha\gamma}}\right)^{\frac{1-\gamma}{\gamma}\frac{\alpha}{1-\alpha}} a_i^{\frac{\alpha}{1-\alpha\gamma}} \exp\left(\varepsilon_i\right)^{\frac{1}{1-\alpha\gamma}} \tag{5}$$

$$C \propto \left(n^{-1}\sum_j a_j^{\frac{1}{1-\alpha\gamma}} \exp\left(\varepsilon_j\right)^{\frac{\gamma}{1-\alpha\gamma}}\right)^{\frac{1-\alpha\gamma}{\gamma}\frac{\alpha}{1-\alpha}} \tag{6}$$

Aggregate output is a CES aggregate over the sector productivities with exponent $\gamma/\left(1-\alpha\gamma\right)$. Since $(1-\alpha\gamma) > 0$, the sign of that exponent depends just on the sign of γ. When the inputs are gross complements ($\gamma < 0$), aggregate output is a complementary function of the sector productivities.

4 Testable predictions of the model

This section examines further predictions of the model. The predictions are divided into three basic categories, all of which take advantage of cross-sector information: the time-series behavior of sector activity; time-variation in the cross-sectional distribution of activity; and conditional covariances across sectors.

4.1 Sector activity in the time series

Log aggregate and sector-level output can be written in the form

$$\log C = \frac{\alpha}{1-\alpha}\frac{1-\alpha\gamma}{\gamma}\log\left(n^{-1}\sum_j a_j^{\frac{1}{1-\alpha\gamma}} \exp\left(\varepsilon_j\right)^{\frac{\gamma}{1-\alpha\gamma}}\right) \tag{7}$$

$$\log Y_i = \frac{1-\gamma}{1-\alpha\gamma}\log C + \frac{1}{1-\alpha\gamma}\varepsilon_i + \text{constants} \tag{8}$$

Prediction 1a: Log sector output is skewed left, but less than aggregate output.

The intuition for that result is simple: sector output is equal to aggregate output plus a symmetrical shock. It is then straightforward to show that the skewness of $\log Y_i$ must be less than that of $\log C$. Due to the additivity, this result holds in both levels and growth rates.

To push this result further, note that the source of all sector skewness here is the exposure to aggregate output. That is because it is fundamentally the aggregation process that causes skewness – the sector shocks themselves are not skewed. So if we extract the sector-specific component of activity, we should not observe any skewness:

Prediction 1b: The residual from a regression of log sector output on log aggregate output is not skewed.

In the model, the regression of $\log Y_i$ on $\log C$ identifies ε_i (up to an affine transformation). Examining the relative skewness of the ε and $\log C$ tests a core prediction of the model, that macro skewness is caused by the aggregation process, rather than by sector- or firm-level skewness. We show below that this allows us to directly test the model against micro-based explanations of skewness, such as the model of Ilut, Kehrig, and Schneider (2018).

4.2 The cross-sectional distribution of activity

There is a long-running literature that studies cross-sectional dispersion in outcomes. This model has strong implications for dispersion. Since aggregate output is a concave function of the sector-level shocks, an increase in their dispersion mechanically reduces sector output, simply through a Jensen's inequality effect. To see that effect, we first have the following result, which follows directly from the definition of the cumulant generating function.

Denote the sample cumulants, weighted by $\left\{a_j^{1/(1-\alpha\gamma)}\right\}$, by $\hat{\kappa}_{a,m}$. The first three sample cumulants are equal to the first three central moments. That is, $\hat{\kappa}_{a,1}(\{\varepsilon_j\})$ is the sample mean of the $\{\varepsilon_j\}$, $\hat{\kappa}_{a,2}(\{\varepsilon_j\})$ is the sample variance, and $\hat{\kappa}_{a,3}(\{\varepsilon_j\})$ is the sample third moment, all weighted by $\left\{a_j^{1/(1-\alpha\gamma)}\right\}$.[6]

[6]Specifically, define the weighted sample mean to be $E_a[\{x_j\}] = n^{-1}\sum_j \frac{a_j^{1/(1-\alpha\gamma)}}{\sum_k a_k^{1/(1-\alpha\gamma)}} x_j$. Then $\hat{\kappa}_{a,1} = E_a[\{\varepsilon_j\}]$, $\hat{\kappa}_{a,2} = E_a\left[\left\{(\varepsilon_j - E_a[\{\varepsilon_k\}])^2\right\}\right]$, and $\hat{\kappa}_{a,2} = E_a\left[\left\{(\varepsilon_j - E_a[\{\varepsilon_k\}])^3\right\}\right]$.

More generally, the nth weighted sample cumulant is the nth derivative with respect to t, evaluated at zero, of the sample cumulant generating function,

$$K_a(t;\{\varepsilon_j\}) \equiv \log \sum_j \frac{a_j^{1/(1-\alpha\gamma)}}{\sum_k a_k^{1/(1-\alpha\gamma)}} \exp(t\varepsilon_j) \tag{9}$$

Proposition 2 *Log aggregate output is*

$$\log C = \frac{\alpha}{1-\alpha} \sum_{m=1}^{\infty} \frac{1}{m!} \left(\frac{\gamma}{1-\alpha\gamma} \right)^{m-1} \hat{\kappa}_{a,m} \left(\{\varepsilon_j\} \right) + \textit{constants} \tag{10}$$

That is, output is linear in the sample cumulants of the set of realized productivity shocks, $\{\varepsilon_j\}$, weighted by the importance of each sector in production. When $\gamma < 0$, the coefficients on the even cumulants are negative, while the coefficients on the odd cumulants are positive. We then immediately have the following predictions

Prediction 2a: The cross-sectional variance of sector activity (or the sector-specific residuals) is countercyclical.

Prediction 2b: The cross-sectional skewness of sector activity (or the sector-specific residuals) is procyclical.

Formally, these predictions follow simply from the fact that when γ is negative, the coefficients on variance and skewness in the cumulant-based representation of output are negative and positive, respectively. That is, all else equal, an increase in cross-sectional variance is associated with lower output, while an increase in cross-sectional skewness (through an increase in the third moment) is associated with an increase in output.

In the model, sector sales shares – the nominal output of each sector divided by total nominal output summed across sectors – are linear functions of sector productivity. That immediately implies that the sales shares have very similar properties to the cross-section of sector activity, yielding the following result:

Prediction 2c: The concentration of sector sales shares is countercyclical.

The countercyclicality of sales shares is closely related to the countercyclicality of volatility in the model described above. Hulten's theorem says that the sensitivity of aggregate output to each sector's technology shock is equal to the sector's sales as a fractions of nominal GDP. When those fractions are more concentrated, the sum of their squares rises, generating higher aggregate volatility.

As above, it should be noted that the cyclicality of the cross-sectional distribution here has nothing to do with changes in conditional distributions or "time-varying uncertainty". The cross-sectional distribution is a random variable that is correlated with output. All the fundamental shocks have constant volatility, and there is no sense in which there are uncertainty shocks here. Proposition 2 shows that there is a mechanical relationship between the cross-sectional sample moments and output.

Finally, note that the predictions here rely on the fact that aggregate output is a function of the sector shocks. If aggregate output were independent of the sector-specific shocks –

i.e. if the sector shocks in some sense "washed out", e.g. if aggregation were linear, so that aggregate output just depended on aggregate shocks – then the cross-sectional distribution of shocks would have no impact on aggregate output.

4.3 Conditional covariances

The model description above does not say anything about the time series dependence of the levels of productivity, ε_i. In order to analyze the conditional covariances we examined empirically, we consider here a version of the model in which the ε_i follow AR(1) processes (though the logic extends to much more general processes).

Proposition 3 *Sector i's covariance with other sectors increases relative to covariances between other sectors when ε_i falls for $\gamma < 0$ (the result is reversed when $\gamma > 0$). Specifically,*

$$sign\left(\frac{d}{d\varepsilon_{i,t}}\begin{bmatrix} \sum_{k\neq i}\text{cov}_t\left(\log Y_{i,t+1}, \log Y_{k,t+1}\right) \\ -\sum_{k\neq j}\text{cov}_t\left(\log Y_{j,t+1}, \log Y_{k,t+1}\right)\end{bmatrix}\right) = sign\left(\gamma\right) \tag{11}$$

We thus have the following prediction.

Prediction 3a: Following a negative innovation in $\varepsilon_{i,t}$, sector i's conditional covariances with other sectors rise relative to other sectors' covariances with each other (this prediction is identical in levels and growth rates, since they have the same conditional distributions up to a level shift).

Prediction 3b: Following the same negative innovation, sector i's conditional covariance with aggregate activity also rises.

Intuitively, proposition 3 and predictions 5 and 6 follow from the fact that when a sector receives a negative shock, it becomes relatively more important in determining variation in aggregate output when $\gamma < 0$. The derivation in the appendix shows that

$$\frac{d}{d\varepsilon_{i,t}}\sum_{j\neq i}\text{cov}_t\left(\log Y_{i,t+1}, \log Y_{j,t+1}\right) = \frac{1-\gamma}{\left(1-\alpha\gamma\right)^2}\left(n-2\right)\text{cov}_t\left(\frac{d}{d\varepsilon_{i,t}}\log C_{t+1}, \varepsilon_{i,t+1}\right) + \begin{matrix}\text{terms} \\ \text{independent of } i\end{matrix} \tag{12}$$

So what matters is how the sensitivity of output to a shock to sector i depends on the current productivity in sector i. According to Hulten's theorem, that depends on the sales share of sector i, which is

$$\frac{n^{-1}a_i^{\gamma/(1-\alpha\gamma)}\exp\left(\varepsilon_i\right)^{\gamma/(1-\alpha\gamma)}}{n^{-1}\sum_j a_j^{\gamma/(1-\alpha\gamma)}\exp\left(\varepsilon_j\right)^{\gamma/(1-\alpha\gamma)}} \tag{13}$$

The sign of the derivative of that ratio with respect to ε_i is the sign of γ. Since all covariation between sectors comes through variation in aggregate output, the relative increase in the importance of the given sector increases its covariances with all other sectors and reduces the covariances of other sectors with each other. Since the sectors are all exposed to aggregate growth, they also inherit heteroskedasticity caused by that effect.

Prediction 3c: The conditional variances of aggregate and sector activity are countercyclical.

The model therefore predicts that uncertainty, as measured by conditional variances, varies with the state of the economy, even though the fundamental shocks are homoskedastic. Furthermore, there is no causal relationship between activity and uncertainty. Intuitively, the mapping from the underlying shocks to output is concave, so its slope is relatively steep when output is low, causing volatility to be high. It is that concavity in the production function that generates both skewness and countercyclical volatility.

Proposition 3 formalizes the idea that under complementarity, when a sector receives a negative shock it becomes more central. It is a core prediction of the model, in that it directly tests the idea that aggregate output is a concave function of the sector shocks. When aggregation is concave, it is exactly the sectors that receive negative shocks that should rise most in importance. We show formally below that most other models of aggregate skewness do not generate such a prediction for time-varying centrality.

4.4 Extensions

Appendix D reports results for two extensions of the baseline model that relax some of the strict assumptions above. First, section D.1 relaxes the assumption that labor is fixed in each sector by allowing elastic labor supply. It shows that the main results continue to hold and that the model can qualitatively match the empirical results both for output and employment. Section D.2 similarly shows that payments to capital in the model share the same features as employment. To the extent that stock returns move with payments to capital (since stock prices are the net present value of those payments), the model can qualitatively generate the empirical results on stock returns.

5 Empirical analysis

This section tests the six empirical predictions of the model developed above. The last subsection discusses additional evidence in favor of the hypothesis that inputs are complements.

5.1 Data

We focus primarily on measures of activity that have data at the monthly frequency or higher and are measured at a high level of sectoral detail. The two monthly series are industrial production (from the Federal Reserve), which is measured to the five-digit NAICS level in manufacturing industries, and employment (from the Current Employment Survey of the BLS), which is measured at up to the six-digit NAICS level and covers the entire economy. For industrial production, we follow Foerster, Sarte, and Watson (2011) and study data since 1972. For employment, the sample with detailed NAICS coverage begins in 1990, while data on two-digit sectors (BLS-defined supersectors) is available since 1972.

We also examine stock returns. Returns have the drawback that they do not directly measure activity, being driven not just by current conditions but also by expectations for the future (in addition to depending on fluctuations in discount rates). However, returns are measured at much higher frequencies – we use up to daily data – which is useful for estimating time-variation in conditional moments. For sector-level measures of stock returns, we construct value-weighted portfolios according to SIC sectors (similar to the portfolios available on Kenneth French's website), requiring at least five firms in a particular sector to include it in the analysis.

5.2 Prediction 1: time-series skewness

Figure 1 documents skewness across levels of aggregation for industrial production, employment, and stock returns. The blue series in the figure, discussed further below, represent estimates of skewness. At a given level of aggregation, we calculate skewness (either for levels or growth rates) in each sector's time series, and then report the (unweighted) mean of those skewness coefficients at each level of aggregation. 90-percent confidence bands are plotted for each estimate.[7]

5.2.1 Prediction 1a: sector skewness is smaller than aggregate skewness

The first and second columns of panels in figure 1 plot estimates of time-series skewness and the aggregate and sector levels for IP, employment, and stock returns, in both growth rates and levels. The blue series are point estimates, while the red series plot the difference between each sector's average skewness and overall aggregate skewness.

[7] The standard errors are calculated with a blockwise jackknife that clusters by date. Specifically, each jackknife replication removes 50 consecutive months of data from the sample – the same 50 months for all sectors – and we iterate over all possible starting months for the excluded dates. See Lahiri (2003).

The top panels report results for industrial production. The skewness of total industrial production growth is -1.18. At the two-digit level – just three sectors: durable and non-durable manufacturing and mining – average skewness is -0.96. At the three- and four-digit levels, where there are 43 and 81 total sectors, respectively, skewness falls to -0.55 then -0.45. Finally, at the five-digit level skewness is only -0.41. Skewness at the aggregate level is therefore three times higher than at the most disaggregated sector level. The red series show confidence bands for those differences, and three of the four are statistically significantly In other words, there is substantially more skewness in industrial production, both in levels and growth rates, at the aggregate than at the disaggregated level, both economically and significantly.

The second and third rows of panels plot results for employment. Again, both are skewed left in levels and growth rates, and sectors are less skewed than the aggregate. The bottom row shows similar results for stock returns. The results for employment are statistically somewhat weaker than for IP, while those for returns are the strongest. Since returns are uncorrelated over time, while employment and IP are substantially serially correlated, the number of effective observations for returns is much larger, increasing statistical power.

Overall, then, the data clearly supports the first prediction of the model, which is that skewness is much greater at the aggregate than at the sector level.

5.2.2 Prediction 1b: sector residuals are unskewed

In the model, sector shocks can be identified from a regression of sector on aggregate activity. Empirically, then, we estimate the regression

$$\Delta y_{i,t} = a_i + b_i \Delta y_t + \nu_{i,t}, \tag{14}$$

where $\Delta y_{i,t}$ denotes the growth rate in some measure of activity in sector i. We also estimate a similar regression in levels. The third and fourth columns of panels in figure 1 report skewness for the residuals $\nu_{i,t}$ as the blue series. The red series plot the difference between skewness for the residuals and skewness for the original data (i.e. $\Delta y_{i,t}$) at the same level of aggregation. The prediction of the theoretical model is that the $\nu_{i,t}$ should be unskewed, since all of the skewness in sector activity in the model is due to exposure to the aggregate component.

Figure 1 shows that the skewness of the residuals is, across all seven panels, economically close to zero. In every case it is less negative than skewness for the original data, and it is typically substantially smaller than -0.5. For the detailed employment data starting in 1990 and the returns data, skewness for residuals is indistinguishable from zero, and in the other two cases it is less than half the magnitude that is observed in the original data.

So while there is consistent evidence for negative skewness when we look at raw growth rates, the residuals are close to symmetrically distributed. Beyond the fact that this result confirms the second prediction of the model, it is also important for helping further demonstrate that skewness originates at the aggregate level. The sector-specific component of activity contributes little to sector asymmetry.

5.3 Prediction 2: the cross-sectional distribution

The second set of predictions of the model is about the cyclicality of the cross-sectional distribution of sector activity. In each month t, we calculate the cross-sectional variance and skewness of monthly growth rates of industrial production and employment at the four-digit NAICS level. We also calculate the mean across days within each month of the variance of stock returns across 140 three-digit SIC sectors on each day.[8] It is important to emphasize that these are *realized* sample moments – they do not measure a conditional distribution, so they do not tell us whether or not the conditional probability density from which the sector growth rates are drawn changes over time. We are simply measuring sample moments – which are random variables – and examining their contemporaneous relationship with the state of the business cycle.

The top panel of table 2 reports results from univariate regressions of the cross-sectional variances of both levels and growth rates of our measures of activity on an NBER recession indicator and aggregate employment growth as two different measures of the state of the business cycle (i.e. in separate regressions). The cross-sectional variance, skewness, and aggregate employment growth are normalized to have unit variance to help in interpreting the coefficients.

Consistent with the predictions of the model, and with past work, we find that there are statistically and economically increases in cross-sectional dispersion when the economy is weak, as measured by either a recession indicator or aggregate employment growth. Across the various estimates, in both levels and growth rates, and for raw measures and sector-specific residuals, variance is on average higher by 0.79 standard deviations during recessions and the correlation with aggregate employment growth is -0.31. Moreover, across the 20 coefficient estimates, the point estimate has the predicted sign in every case (and is statistically significant in all but four). The third prediction of the model, that the cross-sectional

[8] That is, using daily returns in sector i, $r_{i,d}$, the monthly cross-sectional variance is $\frac{1}{\#(days \in t)} \sum_{d \in t} \text{var}_d \left(r_{i,d} \right)$ where var_d is the cross-sectional standard deviation on day d. We use three-digit sectors for stock returns for data availability reasons – there are not enough firms in the CRSP dataset to calculate sector returns with too much detail.

variance of sector activity is countercyclical, has substantial support in the data.

The bottom panel of table 2 reports analogous results for cross-sectional skewness. In this case, we find that skewness is in general procyclical – it is more negative during NBER recessions and positively correlated with aggregate employment growth. That results holds for 19 of the 20 point estimates. The results are statistically somewhat weaker than for variances, but that is not surprising as variances are typically more poorly estimated than variances. On average, the results imply that skewness is more negative by 0.29 standard deviations in recessions and has a correlation of 0.10 with aggregate employment growth. These point estimates are likely biased down due to the measurement error in the monthly estimates of cross-sectional skewness, thus understating the true cyclicality of cross-sectional skewness.

The model's predictions for the time-series behavior of cross-sectional moments therefore have substantial support in the data. Across a variety of measures, both in levels and growth rates, cross-sectional variance is countercyclical and cross-sectional skewness is procyclical. While that result has been found previously when looking at the raw measures of activity, the results here extend those findings by showing that they hold for the sector-specific components, i.e. the sector residuals.

5.4 Prediction 3: Conditional moments

The two sets of results above, on time-series skewness and countercyclical cross-sectional dispersion, can be potentially explained by concavity in f – heuristically, negative second derivatives. When $\partial^2 f/\partial\varepsilon_i^2 < 0$, the common component of activity, y_t, becomes more sensitive to a shock in sector i when that sector's shocks have recently been negative. Since all sectors are exposed to the common factor, when a sector has received a negative shock, $\varepsilon_{i,t} < 0$, it will become more central, in the sense that it will covary more strongly with average activity in other sectors. This section tests that implication of concavity of f. Second, and relatedly, it examines whether the time-series volatility of activity in each sector is countercyclical in the sense of being negatively related to past shocks.

The analysis has two parts. We first examine results for employment and IP. These variables are direct measures of economic activity, but they are only measured at the monthly frequency, making estimation of conditional moments relatively difficult. We next examine stock returns, which have the advantage of high frequency data, allowing for precise estimates of conditional moments, but are also imperfectly linked to activity (as they also depend on changes in expected future growth and discount rates). For the baseline estimates, we use the level of aggregation that has the largest number of sectors – five-digit sectors for employment, four-digit sectors for IP, and three-digit sectors for stock returns.

5.4.1 IP and employment

Empirical method Define Σ_t to be the (unobservable) conditional covariance matrix of sector-level growth rates on date t. Define $\Sigma_{t,i}$ to be the average of the i'th column of Σ_t, excluding the i,i element. $\Sigma_{t,i}$ is the average of the covariances of sector i with all other sectors. Equivalently, it is the covariance of growth in sector i with average growth in all other sectors, which represents the common component of growth rates. When we say that sector i covaries more strongly with other sectors, we mean $\Sigma_{t,i}$ rises.

We also define $\beta_{i,t}$ to be the conditional covariance of activity in sector i with aggregate activity, and $\sigma_{i,t}^2$ to be the conditional variance of sector i. The goal is to estimate relationships of the form

$$\tilde{x}_{i,t} = a_i + \sum_{j=0}^{J-1} b_j \varepsilon_{i,t-j} + c_t + \tilde{\eta}_{i,t} \tag{15}$$

for $\tilde{x}_{i,t}$ equal to $\Sigma_{t,i}$, $\beta_{i,t}$, and $\sigma_{i,t}^2$ and where $\varepsilon_{i,t}$ measures the innovation to the level of activity in sector i on date t and $\tilde{\eta}_{i,t}$ is a residual. The problem is that $\Sigma_{t,i}$, $\beta_{i,t}$, and $\sigma_{i,t}^2$ are not observable. We therefore proxy for them with date-t products, similar to the literature on heteroskedasticity and feasible GLS.

More specifically, define $\varepsilon_{i,t}$ to be the statistical innovation in activity in sector i (i.e. from a forecasting regression).[9] We then proxy for $\sigma_{i,t}^2$ with $\varepsilon_{i,t+1}^2$, $\Sigma_{t,i}$ with $\sum_{j\neq i} \varepsilon_{i,t+1}\varepsilon_{j,t+1}$, $\beta_{i,t}$ with $\varepsilon_{i,t}\varepsilon_{agg,t}$ (where $\varepsilon_{agg,t}$ is the innovation in aggregate activity). Those products are single-observation sample moments when the conditional expectation of $\varepsilon_{i,t}$ is zero, with the property that

$$E_t\left[\sum_{j\neq i} \varepsilon_{i,t+1}\varepsilon_{j,t+1}\right] = \Sigma_{t,i} \tag{16}$$

$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{agg,t+1}\right] = \beta_{i,t} \tag{17}$$

$$E_t\left[\varepsilon_{i,t+1}^2\right] = \sigma_{i,t}^2 \tag{18}$$

[9] Specifically, we forecast activity in each sector using four lags of sector activity and the lagged value of the first three principal components of activity across all sectors.

The regressions that we actually estimate are then

$$
\begin{aligned}
x_{i,t} &= a_{i,1} + \sum_{j=0}^{J-1} b_{1,j} \varepsilon_{i,t-j} + c_{1,t} + \eta^1_{i,t} && (19) \\
\text{where } x_{i,t} &= \sum_{j \neq i} \varepsilon_{i,t+1} \varepsilon_{j,t+1},\ \varepsilon_{i,t+1} \varepsilon_{agg,t+1},\ \text{or } \varepsilon^2_{i,t+1} && (20)
\end{aligned}
$$

$\eta^j_{i,t}$ captures both the true residual, $\tilde{\eta}^j_{i,t}$, and also the measurement error in the dependent variable, $x_{i,t} - \tilde{x}_{i,t}$ (e.g. $\varepsilon^2_{i,t+1} - \sigma^2_{i,t}$). The errors are therefore in general non-Gaussian. Because there may be common components across sectors in the innovations, $\varepsilon_{i,t}$, we include time fixed effects in the estimation and cluster the standard errors by date. Similarly, some sectors will covary with others more strongly on average, so the the estimation includes sector fixed effects. The inclusion of time fixed effects means that changes in the conditional moments are all interpreted as changes relative to those in other sectors. For example, since the date-t mean of $\Sigma_{t,i}$ is equal to the mean of all pairwise covariances, positive values for the $b_{1,j}$ coefficients mean that a positive shock to sector i raises its covariances *relative to those between other sectors.*

Because the $x_{i,t}$ variables are functions of date-$t+1$ observations, the regressions all represent forecasts and hence conditional moments. That is, the fitted value of the right-hand side is a date-t conditional expectation.

Results The top three sections of table 1 report results of the forecasting regressions for IP and employment. In each case, we use whatever level of aggregation yields the largest number of sectors. For IP that is the 4-digit level. For employment, we use 2-digit data that extends to 1972 and 5-digit data since 1990 in separate regressions. In all cases, we use three monthly lags of activity on the right-hand side ($J = 3$) and report the sum of the coefficients in the table. Standard errors clustered by date are reported in brackets.

For the regressions forecasting $\Sigma_{t,i}$, $\beta_{i,t}$, and $\sigma^2_{i,t}$ the coefficients are negative in all three sections. For the first two rows, the coefficients are of similar magnitude, about -0.05, while they are close to zero for the short employment sample (though values close to -0.05 are at the bottom edge of the confidence bands). A value of 0.05 means that when a sector's activity rises by one standard deviation, the product on the left-hand side falls by 0.05 standard deviations. The regressions thus give consistent support to the model's prediction that following a negative shock, a sector becomes more volatile, more central, and more correlated with aggregate activity.

5.4.2 Stock returns

For IP and employment there is only a single observation per month, forcing us to use a single observation to proxy for a covariance, which one might naturally worry would involve a substantial amount of measurement error (though that simply appears in the residual in the regression, and hence is accounted for in the standard errors). Stock returns have the advantage that they are available at the daily frequency and thus allow us to measure the covariance matrix in each month much more accurately. We denote the sample covariance in month t by $\hat{\Sigma}_t$, and then $\hat{\Sigma}_{t,i}$ is the sum of the i'th row, excluding the diagonal element, as above. Similarly, the covariance of each sector's returns with returns on the overall market can be estimated using the daily data within each month, $\hat{\beta}_{i,t}$, and the sector's variance can be estimated from the monthly sample variance. The fourth row of table 3 reports results from the regressions

$$x_{i,t} = a_{i,1} + \sum_{j=1}^{J} b_{1,j} r_{i,t-j} + c_{1,t} + \eta_{i,t}^{1}$$

for $x_{i,t} = \hat{\Sigma}_{t,i}$, $\hat{\beta}_{i,t}$, or $\hat{\sigma}_{i,t}^2$, where $r_{i,t}$ is the return in sector i in month t. That is, we use the same specification as for IP and employment, just replacing $\sum_{j \neq i} \varepsilon_{i,t}\varepsilon_{j,t}$ with $\hat{\Sigma}_{t,i}$, etc.

The results are highly similar to those in the first three rows, with coefficients again close to -0.05. While there is less measurement error in this regression, we also have fewer observations since we use a higher level of aggregation (due to the number of firms available), so the magnitude of the standard errors is similar to that in the first three rows.

5.4.3 TFP shocks

The aim of the regressions above is to test the proposition that a positive shock to a particular sector reduces its covariance with other sectors. They have the advantage of using monthly data, giving them relatively good ability to measure conditional covariances, but they have the drawback that we do not observe technology shocks in those data sets. In this section, we use the monthly NBER-CES manufacturing database, following Acemoglu, Akcigit, and Kerr (2015), which measures productivity at the 6-digit level among manufacturing industries, but is only available at the annual level.

We estimate regressions using the same specification as for IP and employment above (i.e. equations (20) and (19)), but with two modifications. First, we use data on real gross output and hours of production workers instead of IP and employment on the left-hand side. Second, the independent variable, instead of being the lagged statistical innovation in activity, is the lagged statistical innovation in total factor productivity, as reported in the

NBER-CES database.

The bottom two rows of table 3 report the results from this exercise. The results are very similar to what we report for IP and employment above. In particular, for gross output, as with IP (which also measures gross output), the coefficients are close to -0.05 and statistically significant, though in this case the coefficient for sector volatility is close to zero. For hours worked, as with employment, the point estimates are negative, and -0.05 is close to boundary of the confidence bands, but the point estimates themselves are close to zero.

Overall, then, similar to the upper rows of the table, there is strong evidence for gross output following the predictions of the model, in terms of negative shocks increasing covariances, but for employment the results are again weak at best.

5.4.4 Robustness

We have examined a number of robustness tests for the main results. The coefficient estimates are similar when we weight the sectors by their relative size, when we winsorize the dependent variable, and when we estimate the same regressions at alternative levels of aggregation.

6 Quantitative illustration

The model in section 2 is highly stylized, which allowed us to generate simple testable predictions. This section briefly asks whether a quantitative version of the model can match the actual numbers reported in section 3. It is not a full estimation, but rather a simple calibration, showing that the quantitative results are broadly similar to what is produced by a more realistic version of the model. We leave the formidable task of full estimation of a large-scale multisector model to future work.

6.1 Model and calibration

The specification follows that of Baqaee and Farhi (2019) closely (drawing on their posted replication files). The structure of the economy is a general CES setup as in equations (1)–(3) in section 3.1. In this case, the elasticities of substitution between material inputs are the same across sectors – $\gamma = \gamma_i$, but the elasticity of substitution in final output can differ – $\gamma \neq \gamma_C$. Similarly, the elasticity of substitution between labor and materials is fixed across sectors at β. The production parameters $a_{i,j}$, $a_{C,j}$, and α_i are now allowed to vary across sectors. Labor is again normalized to 1. The set of equations becomes

$$
\begin{aligned}
Y_{i,t} &= \exp\left(\varepsilon_{i,t}\right)\left[(1-\alpha_i)+\alpha_i\left(\sum_j a_{i,j}x_{i,j,t}^{\gamma}\right)^{\beta/\gamma}\right]^{1/\beta} \\
C_t &= \left(\sum_j a_{C,j}x_{C,j,t}^{\gamma_C}\right)^{1/\gamma_C}, \quad \sum_i x_{i,j,t}+x_{C,j,t}=Y_{j,t}
\end{aligned}
$$

Our calibration follows that of Baqaee and Farhi (2019) for the most part. The production weights are chosen to match the 1982 input-output table (results are similar for other years). TFP shocks are calibrated to match the relative variance of TFP by industry along with the observed correlations. Their overall scale is chosen to match the volatility of industrial production growth (and the moments we will examine will be matched to the IP data). We draw the shocks from a t distribution with four degrees of freedom, which generates fat tails consistent with observed sector-specific IP growth. The autocorrelation of sector productivity is set to 0.85 (at the monthly frequency) to match the dynamics of sector-level IP growth.

The elasticities of substitution are set to $(\gamma_C, \gamma, \beta) = (0.9, 0.1, 0.5)$. Aggregate output is thus relatively substitutable across sectors – close to a log-linear production function. The strong complementarity arises in production at the sector level, where the assumption is that the mix of material inputs is not amenable to adjustment. These values are similar to those estimated empirically by Atalay (2017).

6.2 Results

Table 4 reports moments of the model corresponding to the results from figure 1 and tables 2–3 along with the associated empirical results for industrial production. The top section shows that, as in the data, skewness is higher at the aggregate than the sector level, though in the model sector skewness is somewhat higher than in the data. Residual and aggregate skewness in the model is well within the empirical confidence bands.

The middle section examines the cyclicality of cross-sectional variance and skewness. NBER recessions in the model are defined as periods when aggregate output growth is in the bottom 15 percent of the unconditional distribution, to replicate the empirical frequency of recessions. The signs and magnitudes of the coefficients are highly similar between the model and the data, both for skewness and variance. The model replicates the empirical result that cross-sectional variance is countercyclical and cross-sectional skewness is procyclical, and the magnitudes are empirically realistic.

Finally, the bottom section reports results for the conditional covariance regressions. Similar to the data, the coefficients in the two versions of the regressions are similar. In the model, they are equal to -0.025, compared to approximately -0.05 in the data. The magnitude of that difference is about the same size as the empirical standard errors, so the data and model again yield quantitatively similar results.

Overall, the quantitative model performs reasonably well in matching the time-series, cross-sectional, and conditional moments, given that we made few choices in the calibration. Table 4 therefore shows that a richer version of the model, which is designed to be closer to quantitative realism than the highly restricted setup analyzed theoretically above, is able to broadly match the empirical behavior of the economy documented in tables 1–3 and figure 1.

7 Alternative models of skewness

This section examines alternative models of skewness. We examine relatively stylized forms of models meant to capture different potential economic mechanisms that could be driving aggregate or cross-sectional skewness. The analysis shows that none of the alternatives is able to generate all of the same predictions – or match the empirical results – developed above.

7.1 Skewed aggregate shocks

Consider the following simple reduced-form specification for aggregate and sector output, y_t and $y_{i,t}$:

$$y_t = \mu_t \tag{21}$$

$$y_{i,t} = b_i y_t + \varepsilon_{i,t} \tag{22}$$

where μ_t is a shock to aggregate output that is skewed left and $\varepsilon_{i,t}$ is symmetrically distributed. This type of reduced-form could be generated by a number of different structural models.[10] The absence of $\varepsilon_{i,t}$ from the equation for y_t can be thought of as resulting from linear aggregation with many sectors, so that the sector-specific shocks average out to zero. The specification in equations (21–22) is a natural way to generate the time-series skewness in aggregate activity documented in table 1. We now examine the ability of the model to match the other features of the data documented in section 3.

[10] Technology or policy shocks, represented by μ_t, could be skewed left (e.g. rare disasters models (Rietz (1988), Barro (2006), Berger, Dew-Becker, and Giglio (2019)). Alternatively, some input to production used by all sectors, e.g. the output of the financial sector (financial intermediation) could be skewed to the left. For example, the financial sector might face occasionally binding constraints (e.g. Brunnermeier and Sannikov (2014), Kocherlakota (2000)).

7.1.1 Time-series skewness

Under the assumption that μ_t is skewed left, aggregate output is also trivially skewed left. Furthermore, it follows from the fact the symmetry of $\varepsilon_{i,t}$ that sector activity is less skewed than aggregate activity. In addition, since the $\varepsilon_{i,t}$ are the residuals of sector output after controlling for the common component, the residuals are unskewed. This model can thus, by assumption, generate the first set of predictions.

7.1.2 Cyclical cross-sectional variance and skewness

On any date, the cross-sectional variance of sector output is

$$\mathrm{var}_t\left(y_{i,t}\right) = \mu_t^2 \,\mathrm{var}\left(b_i\right) + \mathrm{var}_t\left(\varepsilon_{i,t}\right) \tag{23}$$

Since left skewness means that $E\left[\mu_t^3\right] < 0 \Rightarrow corr\left(\mu_t, \mu_t^2\right) < 0$, we immediately have $corr\left(\mathrm{var}_t\left(y_{i,t}\right), y_t\right) \leq 0$. So this model can generate countercyclical volatility as long as b_i differs across sectors (and without stochastic volatility or uncertainty shocks). Recall, though, that the empirical results in table 2 apply not just to sector growth rates, but also to residuals from regressions of sector growth rates on aggregate growth. In this case, that regression identifies $\varepsilon_{i,t}$. If μ_t and $\varepsilon_{i,t}$ are independent, then the cross-sectional moments of the $\varepsilon_{i,t}$ are acyclical.

In other words, then, a model with skewed aggregate shocks can generate countercyclical volatility in sector output, $y_{i,t}$, but it cannot replicate the empirical result that the cross-sectional distribution of residuals from regressions of sector output on aggregate output has the same cyclical properties. So at best the model only get half-way to matching our second empirical fact.

The failure of this model to match the cyclicality of the cross-sectional distribution of the residuals is instructive. The basic structure of this model is superficially similar to the equilibrium of the aggregative model, but it has a critical difference: the common component here is exogenous and independent of the sector-specific shocks, whereas in the aggregative model the common component is endogenous to the sector-specific shocks.

7.1.3 Conditional moments

Under this model, the covariance between any pair of sectors is

$$\mathrm{cov}\left(y_{i,t}, y_{j,t}\right) = b_i b_j \,\mathrm{var}\left(\mu_t\right) + \mathrm{cov}\left(\varepsilon_{i,t}, \varepsilon_{j,t}\right) \tag{24}$$

which, under the model assumptions, is constant. Nothing about the model implies that when a sector shrinks its covariance with other sectors or with aggregate output should rise. Conditional variances of sector growth rates are also constant. Obviously the model could be augmented so that the covariances of the idiosyncratic shocks or the loadings, b, change over time, but there is nothing fundamental about the hypothesis that aggregate shocks are skewed to the left that would require such a scenario. The failure of the model on this dimension is again a result of the independence of the aggregate and sector-specific components.

7.2 Sector output is a concave function of symmetric shocks

Ilut, Kehrig, and Schneider (IKS; 2018) study aggregate and firm-level skewness and argue that skewness can be generated by a model in which firms have concave responses to economic shocks, such that firm output or employment takes the form

$$y_{i,t} = g\left(\mu_t + \varepsilon_{i,t}\right) \tag{25}$$

where μ_t is again an aggregate shock and $\varepsilon_{i,t}$ is an idiosyncratic shock. The shocks are mean-zero and independent with symmetrical distributions. The function g is assumed to be smooth, strictly increasing, and strictly concave. IKS then assume that aggregate output is simply the sum over many firms of $y_{i,t}$ – i.e. it is an expectation across values of $\varepsilon_{i,t}$, conditional on the value of μ_t

$$y_t = E_t\left[y_{i,t}\right] \tag{26}$$

where $E_t\left[\cdot\right]$ denotes an expectation taken across values of i on date t.

Unlike the network model, the model of IKS generates skewness through micro decisions, so it provides a useful benchmark for understanding the difference between skewness arising from concave aggregation of symmetric micro shocks and skewness arising concave micro responses to shocks.

7.2.1 Time-series skewness

It is straightforward to show that sector (or firm) output, $y_{i,t}$, is skewed left, simply due to the concavity of g. However, skewness in this model decreases with the level of aggregation. IKS find this in their simulation (see their table 9). To see why, consider a second-order approximation to sector output, and treat aggregate output as the mean across sectors,

$$y_{i,t} \approx g(0) + g'(0)(\mu_t + \varepsilon_{i,t}) + \frac{1}{2}g''(0)(\mu_t + \varepsilon_{i,t})^2 \tag{27}$$

$$y_t \approx g(0) + g'(0)\mu_t + \frac{1}{2}g''(0)\left(\mu_t^2 + \mathrm{var}(\varepsilon_{i,t})\right) \tag{28}$$

The quadratic term in $y_{i,t}$ is $\frac{1}{2}g''(0)(\mu_t + \varepsilon_{i,t})^2$, while in y_t it is $\frac{1}{2}g''(0)\left(\mu_t^2 + \mathrm{var}(\varepsilon_{i,t})\right)$. The skewness of $y_{i,t}$ is larger than the skewness of y_t essentially because there is more variability in what is being squared at the sector than at the aggregate level – $\mu_t + \varepsilon_{i,t}$ instead of just μ_t.

These results would also obtain in a granular model, as in Gabaix (2011), in which sector shocks are skewed left. That is, suppose there is effectively a small number of sectors, so that the sector shocks have nontrivial effects on aggregate output. Then even if they are skewed, after (linear) aggregation, aggregate output will be less skewed than sector output, due to simple averaging. In other words, the fact that skewness increases with aggregation is inconsistent with a simple form of micro granularity.

7.2.2 Cyclicality of cross-sectional moments

This model naturally generates countercyclical cross-sectional dispersion. Consider a simple linear approximation to sector output around different levels of the aggregate shock,

$$y_{i,t} \approx g(\mu_t) + g'(\mu_t)\varepsilon_{i,t} \tag{29}$$

$$\mathrm{var}_t(y_{i,t}) \approx g'(\mu_t)^2 \mathrm{var}(\varepsilon_{i,t}) \tag{30}$$

By assumption, $g'(\mu_t)$ strictly increases as μ_t declines, making cross-sectional variance countercyclical.

The model also generates countercyclicality for the variance of the sector-specific component of output, as in the data. In the first-order approximation above, the sector-specific component, after removing aggregate output, is $g'(\mu_t)\varepsilon_{i,t}$. The cross-sectional variance of those residuals is then the same as the cross-sectional variance of output itself, and thus has the same cyclicality.

IKS also examine the cyclicality of cross-sectional skewness. They show that cross-sectional skewness is procyclical as long as $g''(x)/g'(x)$ is increasing in x. So under that additional restriction, the model can match our fourth fact.

7.2.3 Sector covariances following negative shocks

Again using a linear approximation, one can show that

$$\mathrm{cov}_t\left(y_{i,t}, y_{j,t}\right) \approx g'\left(\varepsilon_{i,t}\right) g'\left(\varepsilon_{j,t}\right) \mathrm{var}\left(\mu_t\right) \tag{31}$$

$$\mathrm{cov}_t\left(y_{i,t}, y_t\right) \approx g'\left(\varepsilon_{i,t}\right) g'\left(0\right) \mathrm{var}\left(\mu_t\right) \tag{32}$$

When a sector receives a negative shock, $g'\left(\varepsilon_{i,t}\right)$ rises. That raises the covariance of sector i's output with all other sectors and with aggregate output, without having any effect on covariances between other sectors. In that sense, sector i becomes more central. This result is not noted by IKS, but it represents an additional pair of empirical facts that the model matches.

7.2.4 Summary

The model of concave decision rules generates negative skewness, and can match the cross-sectional facts, but it fails to generate the result that skewness increases with the level of aggregation. The key difference between this model and the model of complementarity in production is where skewness arises. With concave decision rules, the skewness arises at the firm or sector level. Under linear aggregation, that skewness washes out (as in the central limit theorem) at the aggregate level. In the network model with complementarity, it is fundamentally *created by* aggregation, leading to a better fit to the data.

7.3 Time-varying uncertainty

A large literature studies models in which shock volatilities change over time. There can be changes in volatility at the aggregate level (Gourio (2012)), idiosyncratic level (Christiano, Motto, and Rostagno (2014)), or both (Bloom (2009)). Such a model, suitably enriched, can potentially also generate skewness. To see why, consider the following reduced-form specification

$$y_t = \sigma_t \mu_t - k\sigma_t^2 \tag{33}$$

$$y_{i,t} = y_t + \sigma_t \varepsilon_{i,t} \tag{34}$$

where y_t, $y_{i,t}$, μ_t, and $\varepsilon_{i,t}$ continue to represent aggregate and sector output and innovations and k is a coefficient determining how output responds to variation in cross-sectional volatility relates to the level of output. We assume here that $\varepsilon_{i,t}$ and μ_t are symmetrically distributed.

7.3.1 Time-series skewness

Time-series skewness can occur in this specification either if shocks to σ_t^2 are skewed to the right – so that $-k\sigma_t^2$ is skewed left, or if μ_t and σ_t are correlated so that the distribution of $\sigma_t\mu_t$ is skewed left. These features are not universal characteristics of models of uncertainty shocks. For example, the model of Bloom et al. (2018) does not generate skewness in aggregate output growth, because both the term $\sigma_t\mu_t$ and changes in volatility are symmetrically distributed. That said, if the conditions necessary for y_t to be skewed left are satisfied, then $y_{i,t}$ is also skewed left, and again by less than y_t, because of the fact that $\varepsilon_{i,t}$ is symmetrically distributed.

7.3.2 Time-varying cross-sectional moments

Cross-sectional variance is time-varying by assumption, since the distribution of $\mu_{i,t}$ has variance σ_{t-1}^2. Cross-sectional variance is also countercyclical for $k > 0$. This result holds both for total sector output and for the sector specific component, $\mu_{i,t}$. This again emphasizes that the key feature of the model to match the cyclicality of cross-sectional moments is that the common component of output cannot be independent of the sector-level residuals.

To generate time-varying cross-sectional skewness, an additional skewness process would need to be added to the model. Salgado, Guvenen, and Bloom (2018) examine such a setup.

Aggregate skewness in the model is determined by the parameter k, and cyclicality of cross-sectional volatility depends on k and m. Procyclical skewness would require a third free parameter. So the model requires a new parameter (or assumption) for each of the empirical results it matches. In that sense it is less parsimonious than the network model, which just requires an assumption about a single parameter (i.e. that inputs be complements).

7.3.3 Conditional moments

The covariance of output between sectors is

$$\operatorname{cov}(Y_{i,t}, Y_{j,t}) = \operatorname{var}(\varepsilon_t) = \sigma_{t-1}^2 \tag{35}$$

In other words, the covariances are all identical. They change over time due to σ_t^2, and they are all countercyclical, but there is no variation across sectors. Certainly nothing in the model requires that when a sector receives a negative shock it will covary more strongly with other sectors, unlike the aggregative model or the one with concave decision rules. In both of those cases, the source of the increased covariance is that a sector loads more heavily on the common component following a negative sector-specific shock. This again illustrates the value of the common component being endogenous to the sector shocks, unlike here, where it is purely exogenous.

7.3.4 Summary

In a model where aggregate output responds negatively to uncertainty, skewness arises naturally and increases with aggregation, as in the data. The model also, by assumption, generates countercyclical cross-sectional and time-series volatility. However, it has no prediction for differences in covariances across sectors. The analysis in this section is based on a reduced-form representation, but it is possible that fully nonlinear solutions of structural models, like that of Bloom et al. (2018), might yield different results. We examined simulations of that model, however (helpfully provided by the authors), and find that output and employment, in both levels and growth rates, are strongly positively skewed, implying that the at least the baseline calibration of Bloom et al. (2018) does not generate the simplest of our empirical results, that levels and growth rates of output and employment are skewed left.

7.4 Implications

The table below summarizes the facts and the ability of the models to qualitatively match them.

Fact:	Model: Complementary network	Skewed shocks	Concave responses	Uncertainty shocks
Increasing skewness with aggregation	✓	✓	×	✓
No skewness for residuals	✓	✓	×	✓
Cyclicality of cross-sectional variance	✓	✓	✓	✓
Cyclicality of cross-sectional resid. var.	✓	×	✓	✓
Centrality rises after negative shocks	✓	×	✓	×

The equilibrium of the network model has two key features that allow it to match the empirical facts: there is skewness in the common but not sector-specific components, and the common component is endogenous to the sector shocks.

8 Conclusion

The goal of this paper is to understand the sources of asymmetries in aggregate output. It studies a network model in which inputs are complements and shows that it naturally generates time-series skewness in aggregate and sector output. It has six additional empirical predictions, all of which have support in the data. The idea of complementarity, advanced most recently by Baqaee and Farhi (2019), is powerful in understanding both the aggregate and cross-sectional behavior of the economy.

The model implies that second and third moments change over time and are cyclical. In the past, it has sometimes been argued that the observed cyclicality of those moments implies that there are exogenous shocks to *uncertainty*, and that uncertainty then has negative effects on the economy. The model advanced here, though, is one in which changes in volatility are a result of fundamental productivity shocks and have no independent effect on the level of output.

A second important implication, which is supported by our empirical contributions, is that the centrality of sectors changes over time. In some models, recessions have common causes, e.g. technology shocks. Here, though, every episode is different. When a sector receives a negative shock, it becomes relatively more important. So in a period where oil stocks are low, shocks to the oil sector become a major driving force (e.g. Hamilton (2003), Kilian (2008)), whereas in periods when the financial sector is highly constrained, financial shocks become most relevant (e.g. Brunnermeier and Sannikov (2014). A key insight of this paper is that complementarity means that the aggregate effects of shocks change in important ways over time, those changes can be measured from the covariances of sector growth rates, and many models fail to match them.

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A Solution of network model

We guess that all sectors, including final consumption, use the same set of inputs (it is simple to confirm that this is true). Define

$$\bar{X} = \left(\sum_j a_j x_j^\gamma \right)^{1/\gamma}$$

with the social optimization to maximize $\bar{X}$. x_j represents the quantity of input j in the bundle of inputs used by all sectors. Sector i's use of input j can then be written as $x_{i,j} = \bar{x}_i x_j$.

The output of sector i is then

$$x_i = z_i \bar{x}_i^\alpha \left(\sum_j a_j x_j^\gamma \right)^{\alpha/\gamma} = z_i \bar{x}_i^\alpha \bar{X}^\alpha \tag{36}$$

Finally, denote the use of good j in final production by cx_j. The resource constraint is then

$$x_j = \sum_i \bar{x}_i x_j + c x_j \tag{37}$$

$$\Rightarrow \quad 1 - c = \sum_i \bar{x}_i \tag{38}$$

Inserting the production function back into the formula for $\bar{X}$, we have

$$\bar{X} = \left(\sum_j a_j z_j^\gamma \bar{x}_j^{\alpha\gamma} \bar{X}^{\alpha\gamma}\right)^{1/\gamma} \tag{39}$$

$$\bar{X}^{1-\alpha} = \left(\sum_j a_j z_j^\gamma \bar{x}_j^{\alpha\gamma}\right)^{1/\gamma} \tag{40}$$

Final consumption can be written as

$$C = \left(\sum_j a_j \left(c x_j\right)^\gamma\right)^{1/\gamma} = c\bar{X} \tag{41}$$

$$= c\left(\left(\sum_j a_j z_j^\gamma \bar{x}_j^{\alpha\gamma}\right)^{1/\gamma}\right)^{1/(1-\alpha)} \tag{42}$$

subject to the constraint that $\sum \bar{x}_j = 1-c$. We first optimize with respect to the $\bar{x}_j$, putting a Lagrange multiplier λ on the constraint,

$$\max \left(\sum_j a_j z_j^\gamma \bar{x}_j^{\alpha\gamma}\right) - \lambda\left(\sum_j \bar{x}_j + c - 1\right) \tag{43}$$

$$\lambda = \alpha\gamma a_j z_j^\gamma \bar{x}_j^{\alpha\gamma-1} \tag{44}$$

$$\bar{x}_j = \left(\lambda^{-1}\gamma\alpha a_j z_j^\gamma\right)^{1/(1-\alpha\gamma)} \tag{45}$$

Summing over j and inserting the constraint yields

$$1-c = \sum_j \left(\lambda^{-1}\gamma\alpha a_j z_j^\gamma\right)^{1/(1-\alpha\gamma)} \tag{46}$$

$$\lambda\alpha^{-1}\gamma^{-1} = \frac{1}{(1-c)^{1-\alpha\gamma}}\left(\sum_j \left(a_j z_j^\gamma\right)^{1/(1-\alpha\gamma)}\right)^{1-\alpha\gamma} \tag{47}$$

and finally

$$\bar{x}_j = (1-c)\left(\sum_j \left(a_j z_j^\gamma\right)^{1/(1-\alpha\gamma)}\right)^{-1} \left(a_j z_j^\gamma\right)^{1/(1-\alpha\gamma)} \tag{48}$$

We end up with

$$\bar{X}^{1-\alpha} = \left(\sum_j a_j z_j^{\gamma} \bar{x}_j^{\alpha\gamma}\right)^{1/\gamma} \tag{49}$$

$$= (1-c)^{\alpha} \left(\sum_j \left(a_j z_j^{\gamma}\right)^{1/(1-\alpha\gamma)}\right)^{1-\alpha\gamma/\gamma} \tag{50}$$

$$\bar{X} = (1-c)^{\frac{\alpha}{1-\alpha}} \left(\sum_j \left(a_j z_j^{\gamma}\right)^{1/(1-\alpha\gamma)}\right)^{\frac{1}{1-\alpha}\frac{1-\alpha\gamma}{\gamma}} \tag{51}$$

Final consumption is

$$c\bar{X} = c\,(1-c)^{\frac{\alpha}{1-\alpha}} \left(\sum_j \left(a_j z_j^{\gamma}\right)^{1/(1-\alpha\gamma)}\right)^{\frac{1}{1-\alpha}\frac{1-\alpha\gamma}{\gamma}} \tag{52}$$

The choice of c does not matter except as a factor of proportionality. In any case, the optimization yields $c = (1-\alpha)$, and hence

$$C = c\bar{X} = (1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}} \left(\sum_j \left(a_j z_j^{\gamma}\right)^{1/(1-\alpha\gamma)}\right)^{\frac{1}{1-\alpha}\frac{1-\alpha\gamma}{\gamma}} \tag{53}$$

Sector output is then

$$Y_i \propto z_i^{1/(1-\alpha\gamma)} a_j^{\alpha/(1-\alpha\gamma)} \left(\sum_j \left(a_j z_j^{\gamma}\right)^{1/(1-\alpha\gamma)}\right)^{\alpha\frac{1-\gamma}{\gamma(1-\alpha)}} \tag{54}$$

B Proposition 2

This section considers a specification in which the ε follow AR(1) processes, but the logic applies in much more general settings. Formally, for all i, $\varepsilon_{i,t} = \phi\varepsilon_{i,t-1} + \mu_{i,t}$.

We want to know the conditional covariance,

$$\mathrm{cov}_t\left(\log Y_{i,t+1}, \log Y_{j,t+1}\right) = \mathrm{cov}_t\left(\frac{1-\gamma}{1-\alpha\gamma}\log C_{t+1} + \frac{1}{1-\alpha\gamma}\varepsilon_{i,t+1}, \frac{1-\gamma}{1-\alpha\gamma}\log C_{t+1} + \frac{1}{1-\alpha\gamma}\varepsilon_{j,t+1}\right) \tag{55}$$

$$= \left(\frac{1-\gamma}{1-\alpha\gamma}\right)^2 \mathrm{var}_t\left(\log C_{t+1}\right) + \frac{1-\gamma}{(1-\alpha\gamma)^2}\mathrm{cov}_t\left(\log C_{t+1}, \varepsilon_{i,t+1}\right) \tag{56}$$

$$+\frac{1-\gamma}{(1-\alpha\gamma)^2}\mathrm{cov}_t\left(\log C_{t+1}, \varepsilon_{j,t+1}\right) \tag{57}$$

We then consider the sum across all other sectors,

$$\begin{aligned}
\sum_{j\neq i}\mathrm{cov}_t\left(\log Y_{i,t+1},\log Y_{j,t+1}\right) &= (n-1)\left(\frac{1-\gamma}{1-\alpha\gamma}\right)^2\mathrm{var}_t\left(\log C_{t+1}\right)+\frac{1-\gamma}{\left(1-\alpha\gamma\right)^2}(n-1)\,\mathrm{cov}_t\left(\log C_{t+1},\varepsilon_{i,t+1}\right) \\
&\quad +\frac{1-\gamma}{\left(1-\alpha\gamma\right)^2}\mathrm{cov}_t\left(\log C_{t+1},\sum_{j\neq i}\varepsilon_{j,t+1}\right) \\
&= (n-1)\left(\frac{1-\gamma}{1-\alpha\gamma}\right)^2\mathrm{var}_t\left(\log C_{t+1}\right)+\frac{1-\gamma}{\left(1-\alpha\gamma\right)^2}(n-2)\,\mathrm{cov}_t\left(\log C_{t+1},\varepsilon_{i,t+1}\right) \\
&\quad +\frac{1-\gamma}{\left(1-\alpha\gamma\right)^2}\mathrm{cov}_t\left(\log C_{t+1},\sum_{j}\varepsilon_{j,t+1}\right)
\end{aligned}$$

Since the empirical regressions include time fixed effects, all that matters is how this sum of covariances differs across i, so we can drop the first and third terms, since they are identical for all i. We are then left with

$$\sum_{j\neq i}\mathrm{cov}_t\left(\log Y_{i,t+1},\log Y_{j,t+1}\right)=\frac{1-\gamma}{\left(1-\alpha\gamma\right)^2}(n-2)\,\mathrm{cov}_t\left(\log C_{t+1},\varepsilon_{i,t+1}\right)+\text{terms independent of } i \tag{62}$$

Now consider the derivative with respect to $\varepsilon_{i,t}$,

$$\begin{aligned}
\frac{d}{d\varepsilon_{i,t}}\sum_{j\neq i}\mathrm{cov}_t\left(\log Y_{i,t+1},\log Y_{j,t+1}\right) &= \frac{1-\gamma}{\left(1-\alpha\gamma\right)^2}(n-2)\frac{d}{d\varepsilon_{i,t}}\mathrm{cov}_t\left(\log C_{t+1},\varepsilon_{i,t+1}\right) && (63)\\
&= \frac{1-\gamma}{\left(1-\alpha\gamma\right)^2}(n-2)\,\mathrm{cov}_t\left(\frac{d}{d\varepsilon_{i,t}}\log C_{t+1},\varepsilon_{i,t+1}\right) && (64)
\end{aligned}$$

We have

$$\begin{aligned}
\log C_{t+1} &= \frac{1-\alpha\gamma}{\gamma}\frac{\alpha}{1-\alpha}\log\left(n^{-1}\sum_j a_j^{\frac{1}{1-\alpha\gamma}}\exp\left(\frac{\gamma}{1-\alpha\gamma}\left(\phi\varepsilon_{j,t}+\mu_{j,t+1}\right)\right)\right) && (65)\\
\frac{d}{d\varepsilon_{i,t}}\log C_{t+1} &= \phi\frac{\alpha}{1-\alpha}\frac{a_j^{\frac{1}{1-\alpha\gamma}}\exp\left(\frac{\gamma}{1-\alpha\gamma}\left(\phi\varepsilon_{j,t}+\mu_{j,t+1}\right)\right)}{n^{-1}\sum_j a_j^{\frac{1}{1-\alpha\gamma}}\exp\left(\frac{\gamma}{1-\alpha\gamma}\left(\phi\varepsilon_{j,t}+\mu_{j,t+1}\right)\right)} && (66)
\end{aligned}$$

It is then straightforward to show that the derivative of $\frac{d}{d\varepsilon_{i,t}}\log C_{t+1}$ with respect to $\mu_{i,t+1}$ has the same sign as γ globally. So when $\gamma<0$, the covariance is negative. Furthermore, $\frac{1-\gamma}{(1-\alpha\gamma)^2}>0$ for $\gamma<1$, so the sign of the sum of covariances is the sign of γ.

C Levels versus growth rates

We analyze in this section a continuous limit of an AR(1) process. We show that changes in a concave function of a set of those processes, $df_t \equiv f(..., \varepsilon_{i,t}, ...) - f(..., \varepsilon_{i,t-dt}, ...)$, are skewed left when the innovations to the underlying ε have fat tails, but not when they are Gaussian (purely diffusive).

We consider an underlying process, which can be thought of as the productivity process analyzed in the paper, that follows an Ornstein–Uhlenbeck process augmented with compound Poisson jumps. Specifically, consider an $\varepsilon_{i,t}$ that follows

$$d\varepsilon_{i,t} = -\phi\varepsilon_{i,t} + \sigma dW_t + k_t dN_t \tag{67}$$

where W_t is a standard Wiener process, N_t a Poisson counting process with intensity λ, and k_t a random variable with a symmetrical distribution.

The solution of the model is such that aggregate output is a function of the sector productivities with the characteristics that $f_i > 0$ and $f_{ii} < 0$ $\forall i$ and $f_{ij} > 0$ $\forall i \neq j$. The question here is under what circumstances df_t is skewed left. That is, when do our results on skewness in levels also apply to first differences?

Now first suppose there are no jumps. Then we can write, somewhat informally,

$$\begin{aligned} df_t &\equiv f(..., \varepsilon_{i,t}, ...) - f(..., \varepsilon_{i,t-dt}, ...) && (68) \\ \varepsilon_{i,t} &= (1 - \phi dt)\, \varepsilon_{i,t-dt} + \sigma dt^{1/2} \mu_{i,t} && (69) \end{aligned}$$

where $\mu_{i,t}$ is a standard Normal random variable.

$$\begin{aligned} df_t &= f\left(..., (1 - \phi dt)\, \varepsilon_{i,t-dt} + \sigma dt^{1/2} \mu_{i,t}, ...\right) - f(x_{t-dt}) && (70) \\ &= \sum_i f_{i,t-dt} \sigma dt^{1/2} \mu_{i,t} + o\left(\sigma dt^{1/2} \varepsilon_t\right) && (71) \end{aligned}$$

where $f_{i,t} = \frac{df_t}{d\varepsilon_{i,t}}$. In the limit as $dt \to 0$, we than have

$$E\left[\left(f(x_t) - f(x_{t-dt})\right)^3\right] = 0 \tag{72}$$

so that the skewness of the changes in $f(x)$ is zero. This follows simply from the smoothness, or local linearity, of f.

Now suppose there are jumps. We have

$$
\begin{aligned}
df_t &= f\left(..., (1-\phi dt)\,\varepsilon_{i,t-dt} + \sigma dt^{1/2}\mu_{i,t} + k_{i,t} dN_{i,t}, ...\right) - f\left(x_{t-dt}\right) && (73)\\
&= \sum_i f_{i,t-dt}\left(\sigma dt^{1/2}\mu_{i,t} + k_{i,t} dN_{i,t}\right) && (74)\\
&\quad + \frac{1}{2}\sum_i\sum_j f_{i,j,t-dt}\left(\sigma dt^{1/2}\mu_{i,t} + k_{i,t} dN_{i,t}\right)\left(\sigma dt^{1/2}\mu_{j,t} + k_{j,t} dN_{j,t}\right) + o\left(k_t^2\right) && (75)
\end{aligned}
$$

$$
\sum_i f_{i,t-dt}\left(\sigma dt^{1/2}\mu_{i,t} + k_{i,t} dN_{i,t}\right) + \frac{1}{2} f_{ii}\left(\sigma^2 dt \mu_{i,t}^2 + k_{i,t}^2 dN_{i,t}^2\right) \tag{76}
$$

The third moment of df_t is the expectation of its cube. That involves taking all third-order combinations of the various terms, such that the expectations are of order dt. It is straightforward to show that all terms involving interactions either between dN or μ terms, μ^j for $j > 2$, or between different i indexes, are of smaller order than dt.

We then have

$$
E\left[df_t^2\right] = \sum_i f_i^2\left(\sigma^2 dt + \text{var}\,(k)\,\lambda dt\right) + \frac{1}{4} f_{ii}^2 \kappa_4 \lambda dt \tag{77}
$$

$$
E\left[df_t^3\right] = \sum_i \left(\frac{3}{2} f_i^2 f_{ii}\kappa_4 + \frac{1}{8} f_{ii}^3 \kappa_6\right)\lambda dt \tag{78}
$$

where $\kappa_j = E\left[k_t^j\right]$.

The skewness is then

$$
\frac{E\left[df_t^3\right]}{E\left[df_t^2\right]^{3/2}} = \frac{\sum_i \left(\frac{3}{2} f_i^2 f_{ii}\kappa_4 + \frac{1}{8} f_{ii}^3\kappa_6\right)\lambda dt}{\left(\sum_i f_i^2\left(\sigma^2 dt + \text{var}\,(k)\,\lambda dt\right) + \frac{1}{4} f_{ii}^2\kappa_4 \lambda dt\right)^{3/2}} < 0 \tag{79}
$$

where the inequality follows from the concavity of f.

We thus have the claimed result that skewness in output growth is zero when the innovations are purely Gaussian but negative when they have fat tails due to jumps.

D Extensions

This section examines three extensions of the model: elastic labor supply, payments to capital, and a multinomial tree structure for the production network.

D.1 Flexible labor

This section shows that the theoretical results for the benchmark model in the main text with fixed labor also hold in a version of the model in which labor is supplied flexibly. The production of the final consumption good and the resource constraints remain the same. The only difference is that now labor can be flexibly chosen at the sector level, so that sector output is

$$Y_i = z_i L_i^{\beta} \left(n^{-1} \sum_j a_j x_{i,j}^{\gamma} \right)^{\alpha/\gamma} \tag{80}$$

with $\beta + \alpha < 1$. In order to be able to generate variation in aggregate employment, the aggregate supply curve for labor cannot be vertical. We therefore consider the opposite case in which the supply curve is flat. That is, we assume that there is a fixed real wage, w, and that labor is elastically supplied at that price. This is obviously an extreme assumption, but it is chosen for simplicity – allowing for an upward-sloping supply curve would be straightforward.[11]

Using the notation from the baseline case, each sector's production is

$$x_i = z_i L_i^{\beta} \bar{x}_i^{\alpha} \left(\sum_j a_j x_j^{\gamma} \right)^{\alpha/\gamma} = z_i L_i^{\beta} \bar{x}_i^{\alpha} \bar{X}^{\alpha} \tag{81}$$

The optimization for the $\bar{x}_j$ terms can be done taking the L_j as given. That means that everywhere that z_i appears in the baseline case, we replace it with $z_i L_i^{\beta}$, yielding, from equation (53),

$$\bar{X} = \alpha^{\frac{\alpha}{1-\alpha}} \left(\sum_j \left(a_j^{1/\gamma} z_j L_j^{\beta} \right)^{\gamma/(1-\alpha\gamma)} \right)^{\frac{1}{1-\alpha} \frac{1-\alpha\gamma}{\gamma}} \tag{82}$$

Now consider maximizing $\bar{X}$ subject to the wage cost for L. The optimization problem and first-order condition are

$$\max_{\{L_j\}} \alpha^{\frac{\alpha}{1-\alpha}} \left(\sum_j \left(a_j z_j^{\gamma} L_j^{\gamma\beta} \right)^{1/(1-\alpha\gamma)} \right)^{\frac{1}{1-\alpha} \frac{1-\alpha\gamma}{\gamma}} - \sum_j w L_j \tag{83}$$

$$w = \alpha^{\frac{\alpha}{1-\alpha}} \frac{\beta}{1-\alpha} \bar{X}^{\frac{1-\gamma}{1-\alpha\gamma}} L_j^{\frac{\gamma\beta - 1 + \alpha\gamma}{1-\alpha\gamma}} \left(a_j z_j^{\gamma} \right)^{\frac{1}{1-\alpha\gamma}} \tag{84}$$

[11] A simple microfoundation for this specification is to assume that agents have utility that is linear and increasing in consumption and concave and decreasing in labor, e.g. $U = C - \chi \frac{L^{1+\eta}}{1+\eta}$. In that case, the first-order condition for labor supply gives $w = \chi L^{\eta}$. When $\eta = 0$, we get the flat labor supply curve. When $\eta > 0$, labor supply slopes up with the wage.

Solving for L_j and then inserting into the formula for $\bar{X}$, we have

$$L_j = \left(w^{-1} \frac{\alpha^{\frac{\alpha}{1-\alpha}} \beta}{1-\alpha} \right)^{\frac{1-\alpha\gamma}{1-\alpha\gamma-\gamma\beta}} \left(\bar{X}^{1-\gamma} a_j z_j^{\gamma} \right)^{\frac{1}{1-\alpha\gamma-\gamma\beta}} \tag{85}$$

$$\bar{X} = \left(w^{-1} \frac{\alpha^{\frac{\alpha}{1-\alpha}} \beta}{1-\alpha} \right)^{\frac{\beta}{1-\alpha-\beta}} \left(\sum_j \left(a_j^{1/\gamma} z_j \right)^{\frac{\gamma}{1-\gamma(\alpha+\beta)}} \right)^{\frac{1-\gamma(\alpha+\beta)}{\gamma} \frac{1}{1-\alpha-\beta}} \tag{86}$$

D.1.1 Aggregate output and employment

The formula for $\bar{X}$ above immediately yields the result that

$$C \propto \left(\sum_j \left(a_j^{1/\gamma} z_j \right)^{\frac{\gamma}{1-\gamma(\alpha+\beta)}} \right)^{\frac{1-\gamma(\alpha+\beta)}{\gamma} \frac{1}{1-\alpha-\beta}} \tag{87}$$

Aggregate labor is

$$\begin{aligned} \sum_j L_j &= \left(w^{-1} \frac{\beta}{1-\alpha} \right)^{\frac{1-\alpha\gamma}{1-\alpha\gamma-\gamma\beta}} \bar{X}^{\frac{1-\gamma}{1-\gamma(\alpha+\beta)}} \sum_j \left(a_j^{1/\gamma} z_j \right)^{\frac{\gamma}{1-\gamma(\alpha+\beta)}} && (88) \\ &\propto \left(\sum_j \left(a_j^{1/\gamma} z_j \right)^{\frac{\gamma}{1-\gamma(\alpha+\beta)}} \right)^{\frac{1-\gamma(\alpha+\beta)}{\gamma} \frac{1}{1-\alpha-\beta}} && (89) \\ &\propto C && (90) \end{aligned}$$

Any results obtained for aggregate output therefore immediately also hold for aggregate labor (up to a factor of proportionality).

The solution for aggregate output here is the same as in the baseline case, but with α replaced by $\alpha+\beta$. Since $\alpha+\beta$ satisfies the same constraint that α does in the baseline case – just that it lies between 0 and 1 – the results for output from the baseline case also hold here.

D.1.2 Sector output and employment

Sector output is (using the formula for $\bar{x}_i$ from (48)

$$\begin{aligned} x_i &\propto z_i \bar{X}^{\frac{(\beta+\alpha)(1-\gamma)}{1-\alpha\gamma-\gamma\beta}} \left(\left(a_j^{1/\gamma} z_j \right)^{\alpha\gamma/(1-\alpha\gamma)} \right) \left(a_j^{1/\gamma} z_j \right)^{\frac{\gamma}{1-\alpha\gamma-\gamma\beta} \beta \frac{1}{1-\alpha\gamma}} \\ &\propto C^{(\beta+\alpha)\frac{(1-\gamma)}{1-\gamma(\alpha+\beta)}} z_i^{\frac{1}{1-\gamma(\alpha+\beta)}} a_j^{\frac{(\alpha+\beta)}{1-\alpha\gamma-\gamma\beta}} \end{aligned}$$

This again is the same as in the baseline case but with α replaced by $\alpha+\beta$.

Sector employment is

$$
\begin{aligned}
L_j &= \left(w^{-1}\frac{\alpha^{\frac{\alpha}{1-\alpha}}\beta}{1-\alpha}\right)^{\frac{1-\alpha\gamma}{1-\alpha\gamma-\gamma\beta}}\left(\bar{X}^{1-\gamma}a_j z_j^{\gamma}\right)^{\frac{1}{1-\alpha\gamma-\gamma\beta}} && (91)\\
&\propto C^{\frac{1-\gamma}{1-\gamma(\alpha+\beta)}}\left(a_j z_j^{\gamma}\right)^{\frac{1}{1-\gamma(\alpha+\beta)}} && (92)
\end{aligned}
$$

There is a factor structure in employment, as in output. The only difference is that the exponent on z_j is multiplied by γ. As with output, cross-sectional dispersion in employment measures cross-sectional dispersion in productivity, so that it is also negatively related to aggregate output.

D.1.3 Comovement

We next examine comovement.

$$
\begin{aligned}
\operatorname{cov}\left(\log L_j,\log\sum_i L_i\right) &= \operatorname{cov}\left(\log L_j,\log\bar{X}\right) && (93)\\
&= \operatorname{cov}\left(\log\bar{X},\log\left(\bar{X}^{\frac{1-\gamma}{1-\gamma(\alpha+\beta)}}z_j^{\frac{\gamma}{1-\gamma(\alpha+\beta)}}\right)\right) && (94)\\
&= \frac{1-\gamma}{1-\gamma(\alpha+\beta)}\operatorname{var}\left(\log\bar{X}\right)+\frac{\gamma}{1-\gamma(\alpha+\beta)}\operatorname{cov}\left(\log\bar{X},\varepsilon_j\right) && (95)
\end{aligned}
$$

The first term is nonnegative, while the sign of the second term depends on the sign of γ.

$$
\begin{aligned}
\frac{d\bar{X}}{d\varepsilon_i} &= \left(\sum_j\left(a_j^{1/\gamma}\exp\varepsilon_j\right)^{\frac{\gamma}{1-\gamma(\alpha+\beta)}}\right)^{\frac{1-\gamma(\alpha+\beta)}{\gamma}\frac{1}{1-\alpha-\beta}-1}\frac{1-\gamma(\alpha+\beta)}{\gamma}\frac{1}{1-\alpha-\beta}\frac{\gamma}{1-\gamma(\alpha+\beta)}z_j^{\frac{\gamma}{1-\gamma(\alpha+\beta)}-1} && (96)\\
&\propto \frac{1}{1-\alpha-\beta}z_j^{\frac{\gamma}{1-\gamma(\alpha+\beta)}-1} && (97)
\end{aligned}
$$

This derivative is positive. That says that $\text{cov}\left(\log \bar{X}, \varepsilon_j\right) > 0$. Furthermore, it is decreasing in z_j when there is complementarity because the exponent is negative for $\gamma < 0$.

$$\begin{aligned}
\frac{\gamma}{1-\gamma(\alpha+\beta)} - 1 &< 0 && (98)\\
\frac{\gamma}{1-\gamma(\alpha+\beta)} &< 1 && (99)\\
\gamma &< \frac{1}{1+\alpha+\beta} && (100)
\end{aligned}$$

So the derivative is decreasing for $\gamma < \frac{1}{1+\alpha+\beta}$, which includes $\gamma < 0$. Now suppose employment in sector j falls relative to other sectors. That means that ε_j rose. That makes the term $\text{cov}\left(\log \bar{X}, \varepsilon_j\right)$ smaller. To then the term $\frac{\gamma}{1-\gamma(\alpha+\beta)} \text{cov}\left(\log \bar{X}, \varepsilon_j\right)$ is less negative. That is, it rises. We therefore again get the result that a decline in employment in some sector raises its covariance with aggregate employment.

D.2 Payments to capital

There are two ways to see what will happen to profits. The first is a mathematical argument. We know that payments to labor and capital are proportional to each other. Furthermore, payments to labor are proportional to labor itself, since the wage is fixed. That means that payments to capital are also proportional to labor. So any results above for labor must also hold for payments to capital.

The alternative is to solve for prices and payments to capital directly.

Following standard methods, prices are

$$\begin{aligned}
p_i &= C^{1-\gamma} n^{-1} a_i x_{C,i}^{\gamma-1} && (101)\\
&= \bar{X}^{1-\gamma} a_i x_i^{\gamma-1} && (102)
\end{aligned}$$

The total revenue of sector i is then

$$\begin{aligned}
p_i x_i &= \bar{X}^{1-\gamma} a_i x_i^{\gamma} && (103)\\
&= \bar{X}^{1-\gamma} a_i a_i^{\alpha\gamma/(1-\alpha\gamma)} z_i^{\gamma/(1-\alpha\gamma)} \bar{X}^{\alpha\gamma\frac{1-\gamma}{1-\alpha\gamma}} && (104)\\
&= \bar{X}^{\frac{1-\gamma}{1-\alpha\gamma}} a_i^{1/(1-\alpha\gamma)} z_i^{\gamma/(1-\alpha\gamma)} && (105)
\end{aligned}$$

This is the same as the formula for labor from above, except with $(\alpha+\beta)$ replaced by just α.

We therefore have that the behavior of payments to capital – dividends – is the same as for labor. Since this is a purely static model, the only direct mapping to stocks is that a stock is a claim on dividends in the single period of the model.

E Results on model of concave responses

Suppose sector and aggregate output are

$$Y_{i,t} = f\left(\varepsilon_t + \mu_{i,t}\right) \tag{106}$$
$$Y_t = \int_i Y_{i,t} \tag{107}$$

We can approximate these, assuming symmetric fundamental shocks, as

$$Y_{i,t} \approx f(0) + f'(0)\left(\varepsilon_t + \mu_{i,t}\right) + \frac{1}{2}f''(0)\left(\varepsilon_t + \mu_{i,t}\right)^2 \tag{108}$$
$$Y_t \approx f(0) + f'(0)\,\varepsilon_t + \frac{1}{2}f''(0)\,\varepsilon_t^2 + f'(0)\int_i \mu_{i,t} + \frac{1}{2}f''(0)\,2\varepsilon_t \int_i \mu_{i,t} + \frac{1}{2}f''(0)\int_i \mu_{i,t}^2 \tag{109}$$
$$= f(0) + f'(0)\,\varepsilon_t + \frac{1}{2}f''(0)\left(\varepsilon_t^2 + \sigma_\mu^2\right) \tag{110}$$

E.0.1 Lemma for skewness

First, a lemma. We are going to examine random variables of the form

$$x = \frac{a\sigma^2}{2}\left(\frac{\varepsilon}{\sigma}\right)^2 + b\sigma\frac{\varepsilon}{\sigma} + c \tag{111}$$

This has the form of a non-central χ^2. Specifically,

$$x = \frac{a}{2}\sigma^2\left(\frac{\varepsilon}{\sigma_\varepsilon} + \frac{b}{a\sigma}\right)^2 - \frac{1}{2}\frac{b^2}{a} + c \tag{112}$$
$$\sim -\frac{1}{2}\frac{b^2}{a} + c + \frac{a}{2}\sigma^2\chi^2(1,\lambda) \tag{113}$$

where $\chi^2(1,\lambda)$ is a non-central χ^2 with one degree of freedom and noncentrality parameter λ, where in this case $\lambda = \left(\frac{b}{a\sigma}\right)^2$. The skewness of x is then

$$skew(x) = sign\left(\frac{a}{2}\sigma^2\right)2^{3/2}\frac{k+3\lambda^2}{\left(k+2\lambda^2\right)^{3/2}} \tag{114}$$
$$\frac{dskew(x)}{d\lambda} = -sign\left(\frac{a}{2}\sigma^2\right)\frac{3\lambda}{(1+2\lambda)^{5/2}} \tag{115}$$

Finally,

$$\begin{aligned}
\frac{dskew(x)}{d\sigma} &= \frac{dskew(x)}{d\lambda}\frac{d\lambda}{d\sigma} && (116)\\
&= -2\frac{dskew(x)}{d\lambda}\left(\frac{b}{a}\right)^2\sigma^{-3} && (117)
\end{aligned}$$

which implies

$$\begin{aligned}
sign\left(\frac{dskew(x)}{d\sigma}\right) &= -sign\left(\frac{dskew(x)}{d\lambda}\right) && (118)\\
sign\left(\frac{dskew(x)}{d\sigma}\right) &= sign(a) && (119)
\end{aligned}$$

E.1 Aggregate and sector skewness

Both aggregate and sector output take the form

$$x = \frac{a\sigma^2}{2}\left(\frac{\varepsilon}{\sigma}\right)^2 + b\sigma_\varepsilon\frac{\varepsilon}{\sigma} \qquad (120)$$

from above. Specifically, for aggregate output

$$\begin{aligned}
Y_t &= \frac{1}{2}\sigma_\varepsilon^2 f''(0)\frac{\varepsilon_t^2}{\sigma_\varepsilon^2} + f'(0)\sigma_\varepsilon\frac{\varepsilon_t}{\sigma_\varepsilon} + f(0) + \frac{1}{2}f''(0)\sigma_\mu^2 && (121)\\
a &= f''(0),\ b = f'(0),\ \text{and } \sigma^2 = \sigma_\varepsilon^2 && (122)
\end{aligned}$$

and for sector output

$$\begin{aligned}
Y_{i,t} &= \frac{1}{2}f''(0)\left(\sigma_\varepsilon^2+\sigma_\mu^2\right)\frac{\left(\varepsilon_t+\mu_{i,t}\right)^2}{\left(\sigma_\varepsilon^2+\sigma_\mu^2\right)} + f'(0)\sqrt{\sigma_\varepsilon^2+\sigma_\mu^2}\frac{\left(\varepsilon_t+\mu_{i,t}\right)}{\sqrt{\sigma_\varepsilon^2+\sigma_\mu^2}} + f(0) && (123)\\
a &= f''(0),\ b = f'(0),\ \text{and } \sigma^2 = \sigma_\varepsilon^2+\sigma_\mu^2 && (124)
\end{aligned}$$

so they have the same form except for differences in the variance – σ_ε^2 versus $\sigma_\varepsilon^2+\sigma_\mu^2$. We can therefore apply the result from above, Since the σ^2 for sector output is greater than the σ^2 for aggregate output, and since $sign(a) < 0$, $skew(Y_{i,t}) < skew(Y_t)$. That is, sector skewness is more negative than aggregate skewness. Intuitively, the curvature in f has greater relevance when the shocks are larger – for very small shocks, f is effectively linear and induces no asymmetry in the distribution. So in general this model predicts aggregate skewness is smaller than sector skewness.

Table 1. Measures of aggregate time-series skewness

Growth rates

	Skewness		Tail Probability ratios				
		SD cutoffs:	1	1.5	2	2.5	3
IP	-1.22		1.06	1.04	1.75	3.33	9.00
	[0.085]		[0.7032]	[0.819]	[0.291]	[0.111]	[0.0918]
Employment	-0.53		1.20	1.74	3.43	6.00	4.00
	[0.249]		[0.3164]	[0.09]	[0.0738]	[0.155]	[0.4298]
Returns	-0.57		1.08	1.56	1.50	2.00	4.00
	[0.0646]		[0.3266]	[0.0174]	[0.3414]	[0.2396]	[0.2082]
GDP	-0.36		0.92	1.30	2.00	3.00	2.00
	[0.4628]		[0.7094]	[0.4327]	[0.3208]	[0.4381]	[0.7078]
Consumption	-0.79		0.74	0.92	2.33	∞	∞
	[0.1365]		[0.1259]	[0.8394]	[0.1436]	[0.1646]	[0.533]
Investment	-0.86		0.79	1.30	1.17	∞	∞
	[0.1574]		[0.1617]	[0.304]	[0.7581]	[0.238]	[0.5339]

Levels

	Skewness		Tail Probability ratios				
		SD cutoffs:	1	1.5	2	2.5	3
IP	-0.95		0.79	1.78	∞	∞	∞
	[0.0794]		[1.6998]	[0.2802]	[0.088]	[0.3408]	[0.483]
Employment	-0.68		1.32	1.81	3.83	∞	∞
	[0.126]		[0.3852]	[0.3688]	[0.4168]	[0.5232]	[0.4536]
Returns	-1.13		1.01	3.29	∞	∞	∞
	[0.0376]		[0.9878]	[0.0194]	[0.0484]	[0.2726]	[0.4082]
GDP	-0.45		0.83	1.75	8.00	∞	
	[0.1905]		[0.45]	[0.289]	[0.2214]	[0.5818]	
Consumption	-0.45		1.30	1.56	6.00	∞	
	[0.1391]		[0.2838]	[0.4661]	[0.4439]	[0.4439]	
Investment	-1.20		0.96	5.00	∞	∞	
	[0.0638]		[0.8849]	[0.0238]	[0.1328]	[0.4407]	

Table 2a. Correlation of cross-sectional moments with the business cycle – levels

	IP level	IP level residuals	Employment	Employment residuals
Variance				
NBER rec. ind.	0.75 ***	1.00 ***	0.66	1.18
	[0.20]	[0.27]	[0.41]	[0.73]
Agg. empl. growth	-0.23 ***	-0.30 ***	-0.32 ***	-0.57 ***
	[0.07]	[0.10]	[0.11]	[0.20]
Skewness				
NBER rec. ind.	0.09	0.08	-0.38 ***	-1.21 ***
	[0.23]	[0.22]	[0.08]	[0.27]
Agg. empl. growth	0.00	0.00	0.13 ***	0.41 ***
	[0.07]	[0.07]	[0.03]	[0.11]
# of obs.	566	566	352	352

Table 2b. Correlation of cross-sectional moments with the business cycle – growth rates

	IP growth	IP growth residuals	Employment growth	Employment growth residuals	Returns	Return residuals
Variance						
NBER rec. ind.	0.48 ***	0.41 ***	0.95 ***	0.56 ***	1.00 **	1.00 **
	[0.16]	[0.14]	[0.23]	[0.16]	[0.41]	[0.41]
Agg. empl. growth	-0.14	-0.11	-0.39 ***	-0.28 ***	-0.36 **	-0.36 **
	[0.09]	[0.08]	[0.06]	[0.07]	[0.15]	[0.15]
Skewness						
NBER rec. ind.	-0.23 **	-0.02	-0.44 ***	-0.400 ***	-0.17	-0.16
	[0.09]	[0.10]	[0.13]	[0.14	[0.16]	[0.16]
Agg. empl. growth	0.07 *	0.00	0.13 ***	0.11 **	0.05	0.05
	[0.04]	[0.05]	[0.05]	[0.05]	[0.06]	[0.06]
# of obs.	566	566	352	352	588	588

Notes: Employment growth and cross-sectional variance are standardized to have unit over time. Each cell is the coefficient from a univariate regression. Standard errors, reported in brackets, are calculated by Newey–West with 12 monthly lags. The columns labeled residuals use the cross-sectional variance of residuals from regressions of sector growth rates on aggregate growth. * indicates significance at the 10-percent level, ** 5 percent, and *** 1 percent.

Table 3. conditional covariance regression results

	$\Sigma_{t,i}$			$\beta_{i,t}$			$\sigma^2_{i,t}$		
IP (4-digit)									
	-0.047	[0.02]	**	-0.053	[0.021]	***	-0.067	[0.022]	***
Employment (1972–2019; 2-digit)									
	-0.109	[0.046]	**	-0.062	[0.039]	*	-0.113	[0.053]	**
Employment (1990–2019; 5-digit)									
	-0.004	[0.017]		-0.014	[0.016]		-0.001	[0.019]	
Stock returns (3-digit)									
	-0.059	[0.022]	***	-0.075	[0.027]	***	-0.105	[0.043]	**
Manufacturing (6-digit)									
Shipments	-0.043	[0.018]	**	-0.04	[0.019]	**	0.01	[0.011]	
Hours	-0.010	[0.014]		-0.012	[0.014]		-0.02	[0.012]	*

Notes: Each row reports results of regressions measuring the response of conditional covariances to lagged innovations. For each data source, we use whichever level of aggregation yields the largest number of sectors. The left-hand column reports results where the dependent variable is the covariance of each sector's growth rate with the sum of those for all other sectors. The middle column reports results for covariances with aggregate growth rates, and the right-hand column sector volatility. For the first four rows, the independent variable is the lagged statistical innovation in the sector's growth rate. In the bottom section it is the lagged statistical innovation in TFP. All regressions include time and sector fixed effects and standard errors are clustered by date. The first four rows use monthly data and report the sum of the coefficients on three monthly lags. The final two rows have annual data and report the coefficient on a single annual lag.

Table 4. Model simulation results

Skewness

	Model	Data	Data std. err.
Aggregate	-1.72	-1.25	[0.68]
Sector	-0.94	-0.42	[0.20]
Residual	-0.17	-0.27	[0.15]

Cyclicality of cross-sectional moments

	Growth rates:			Residuals:		
	Model	Data	Data std. err.	Model	Data	Data std. err.
Variance:						
Rec. ind.	0.61	0.48	[0.16]	0.40	0.41	[0.14]
ΔIP	-0.24	-0.14	[0.09]	-0.17	-0.11	[0.08]
Skewness:						
Rec. ind.	-0.20	-0.23	[0.09]	-0.07	-0.02	[0.10]
ΔIP	0.10	0.07	[0.04]	0.03	0	[0.05]

Conditional covariance regressions

	Sigma(i,t)			beta(i,t)		
	Model	Data	Data std. err.	Model	Data	Data std. err.
Sum of Coefficients:	-0.026	-0.047	[0.02]	-0.021	-0.053	[0.02]

Notes: The "model" columns report moments from a simulation of 10,000 periods from the numerical solution to the model. The "data" columns report corresponding empirical estimates and standard errors. The sector-level estimates in the top and bottom sections are for IP for 4-digit industries.

Figure 1. Time-series skewness