How risky is consumption in the long-run? Benchmark estimates from a novel unbiased and efficient estimator

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Introduction

- A large literature studies how risky the economy is
- For Epstein–Zin preferences, correct measure is *long-run standard deviation* (LRSD) of consumption growth
- Estimating the LRSD is difficult
- This paper:
  - Develops novel non-parametric estimator
  - Estimates LRSD with data back to 1834

Epstein–Zin preferences:

$$V_t = \left\{ (1-\beta)\, C_t^{1-\rho} + \beta E_t \left[V_{t+1}^{1-\alpha} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$

ρ : inverse EIS
α: risk aversion

Pricing kernel

- Price of risk (through the HJ bound) depends on volatility of the SDF
- Assume log-normal, homoskedastic consumption growth
- Standard deviation of the SDF:

$$std\left(M_{t+1}\right) \approx std\left(\rho \Delta E_{t+1} \Delta c_{t+1} + \left(\alpha - \rho\right) \Delta E_{t+1} \sum_{j=0}^{\infty} \beta^{j} \Delta c_{t+1+j}\right)$$

(exact with unit EIS)

Δc_t: log consumption growth

Pricing kernel

Let $\beta \rightarrow 1$

$$std\left(M_{t+1}\right) \approx std\left(\rho\Delta E_{t+1}\Delta c_{t+1} + (\alpha - \rho)\underbrace{\Delta E_{t+1}\sum_{j=0}^{\infty}\Delta c_{t+1+j}}_{LRSD}\right)$$

- News about $\sum_{j=0}^{\infty}\Delta c_{t+1+j}$ is news about $c_{t+\infty}$
- Most calibrations: $\alpha \gg \rho$

$$std\left(M_{t+1}\right) \approx \alpha \times LRSD$$

- Implies the long-run component dominates
    - Long-run risk model is about making $\Delta E_{t+1}\sum_{j=0}^{\infty}\theta^{j}\Delta c_{t+1+j}$ very volatile
- *LRSD* is key to calibrating any model with Epstein–Zin preferences

Recent calibrations

Table 1. Recent calibrations of the long-run standard deviation of consumption growth (annualized)

	Long-run SD	Moments matched
Campbell and Cochrane (1999)	1.50	SD(dc) 1947-1995
Gourio (2010)	2.00	SD(TFP), 1947-2010
Barro(2006), Wachter (2010)	2.00	SD(dy) 1954-2004, international
Tallarini (2000)	2.30	SD(dc), 1948-1993
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Bansal, Kiku, and Yaron (2008)	4.54	Annual SD(dc), autocorrelations, 1929-2008
Drechsler and Yaron (2011)	4.83	Annual SD(dc), autocorrelations, 1929-2006
Campanale, Castro, and Clementi (2010)	5.20	SD(dy), 1947-2005
Bansal and Yaron (2004); Croce, Lettau, and Ludvigson (2010)	6.28	Annual SD(dc), autocorrelations, 1929-1998
Croce (2010)	8.05	Annual SD(dTFP), 1947-2010
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Long-run standard deviation

- LRSD appears frequently in econometrics:
  - LRSD is the std. dev. of innovations to the Beveridge–Nelson trend (martingale component of c_t)
  - LRSD determines standard errors in OLS and GMM (e.g. Newey–West estimator)
  - Square root of spectral density at frequency zero
- Large literature on estimating LRSD

Smoothed periodogram

- Spectral density is $f(\omega)$

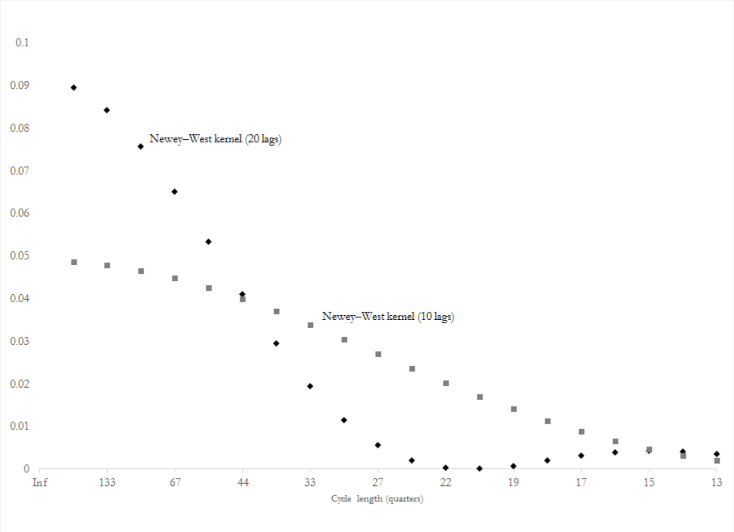
$$LRSD = \sqrt{f(0)}$$

Need to estimate $f(0)$

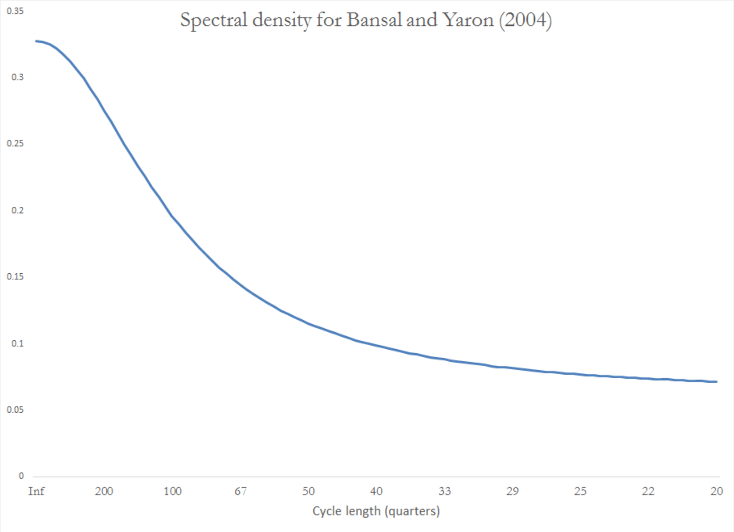
- Periodogram is the sample spectrum
  - Defined only at $T-1$ frequencies
  - Measured with error
- Smoothed periodogram estimator:

$$\hat{f}(0) = \sum_{k=0}^{T-1} K(\omega_k)\, p(\omega_k)$$

Spectral Kernels



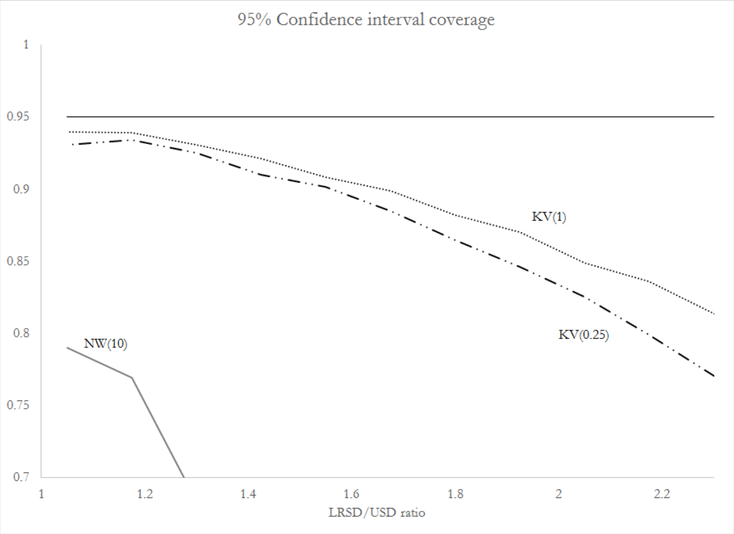
Benchmark model has strongly peaked spectrm



Bias and variance

$$\begin{aligned} bias &\approx \frac{1}{2} f''(0) \int_{-\pi}^{\pi} \omega^2 K(\omega)\, d\omega \\ variance &\approx \frac{4\pi}{T} f(0)^2 \int_{-\pi}^{\pi} K(\omega)^2\, d\omega \end{aligned}$$

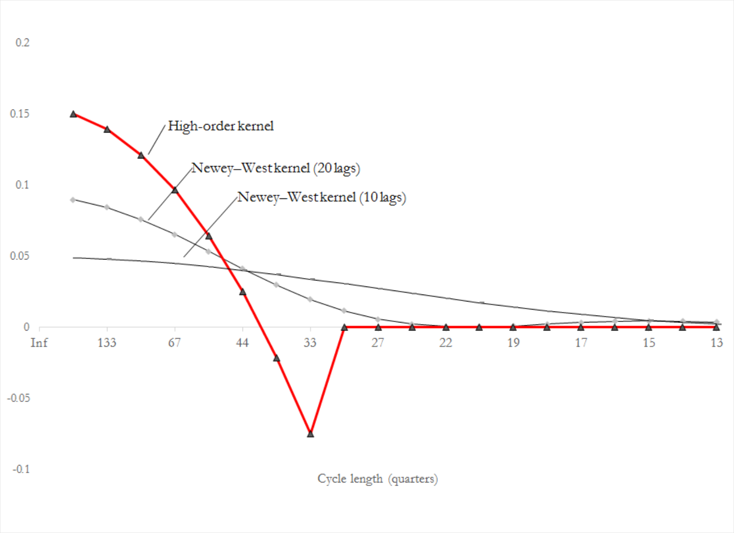
- More peaked kernel:
  - Reduces bias
  - Increases variance
- Changing NW lag length moves along bias/variance tradeoff
- Can we expand the frontier? Yes.

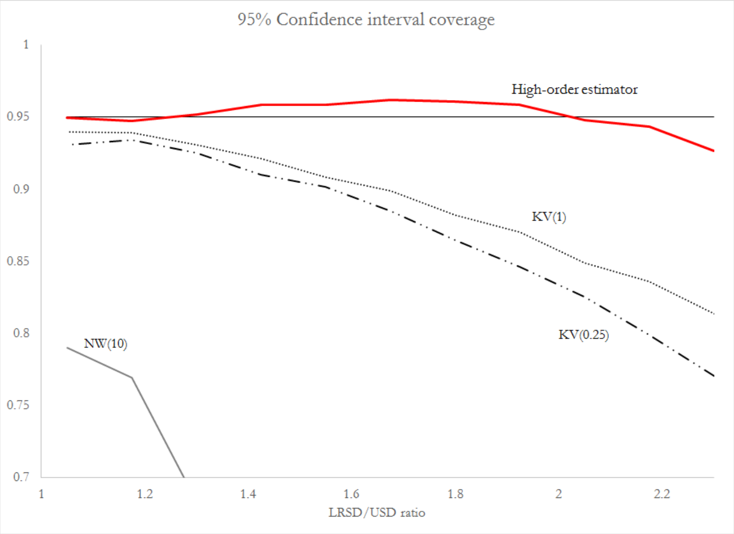

95% Confidence interval coverage
1
0.95
0.9
0.85
0.8
0.75
0.7
1
1.2
1.4
1.6
1.8
2
2.2
LRSD/USD ratio
KV(1)
KV(0.25)
NW(10)

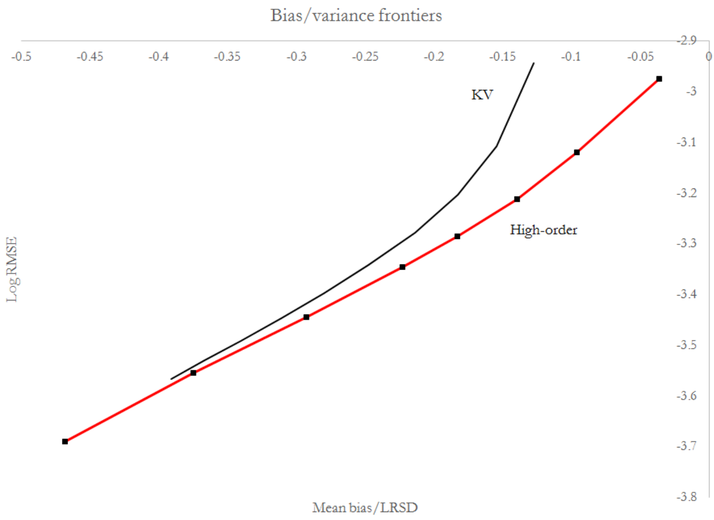
Bias and variance

$$\begin{aligned} bias &\approx \frac{1}{2} f''(0) \int_{-\pi}^{\pi} \omega^2 K(\omega)\, d\omega \\ variance &\approx \frac{4\pi}{T} f(0)^2 \int_{-\pi}^{\pi} K(\omega)^2\, d\omega \end{aligned}$$

- If $K(\omega)$ can be negative, can set approx. bias to zero
- This paper:
  - Set bias to zero
  - Minimize variance
  - Similar to Epanechnikov kernel
- Call it the "high-order kernel"
  - Can then extrapolate to low frequencies
  - Yields lower bias given variance

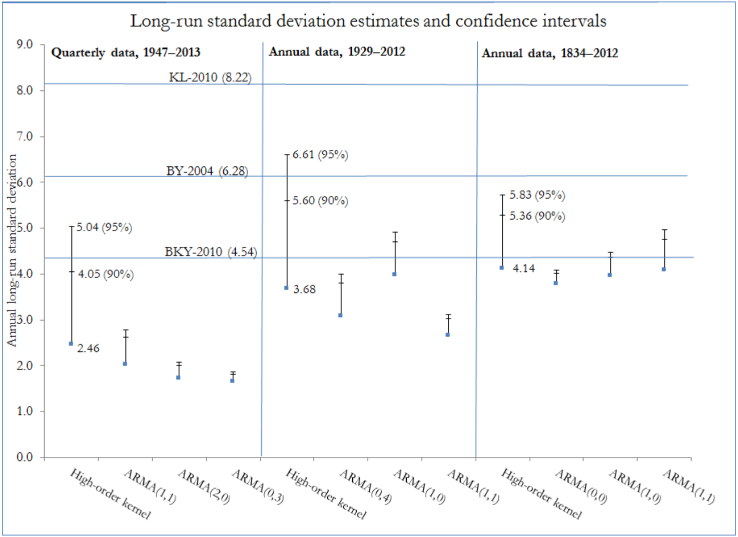

0.2
0.15
0.1
0.05
0
-0.05
-0.1
High-order kernel
Newey–West kernel (20 lags)
Newey–West kernel (10 lags)
Inf
133
67
44
33
27
22
19
17
15
13
Cycle length (quarters)


95% Confidence interval coverage
1
0.95
0.9
0.85
0.8
0.75
0.7
1
1.2
1.4
1.6
1.8
2
2.2
LRSD/USD ratio
High-order estimator
KV(1)
KV(0.25)
NW(10)


Bias/variance frontiers
-0.5
-0.45
-0.4
-0.35
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
-2.9
-3
-3.1
-3.2
-3.3
-3.4
-3.5
-3.6
-3.7
-3.8
KV
High-order
Log RMSE
Mean bias/LRSD

- High-order estimator yields:
    - Almost exact CI coverage
    - Superio bias/variance tradeoff

- Now apply high-order estimator to the data
- Three samples:
    - Post-war quarterly
    - Post-1929 annual
    - Post-1834 annual (Barro and Ursua)



Long-run standard deviation estimates and confidence intervals
Quarterly data, 1947–2013
Annual data, 1929–2012
Annual data, 1834–2012
KL-2010 (8.22)
BY-2004 (6.28)
BKY-2010 (4.54)
5.04 (95%)
4.05 (90%)
2.46
6.61 (95%)
5.60 (90%)
3.68
5.83 (95%)
5.36 (90%)
4.14
Annual long-run standard deviation
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
High-order kernel
ARMA(1,1)
ARMA(2,0)
ARMA(0,3)
High-order kernel
ARMA(0,4)
ARMA(1,0)
ARMA(1,1)
High-order kernel
ARMA(0,0)
ARMA(1,0)
ARMA(1,1)

- Full-sample point estimate: 4.14% per year
- Post-war data much less volatile
- Conservative LRR calibrations look reasonable
- Parametric estimators yield *much* tighter CI

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Figure 4. Rolling LRSD estimates, 60-year window
LRSD 90-percent confidence interval
LRSD point estimate
Long-run standard deviation (Log scale)
16
8
4
2
1
1834
1844
1854
1864
1874
1884
1894
1904
1914
1924
1934
1944
Starting date of 60-year window

Conclusion

- Long-run standard deviation is key moment for models with Epstein–Zin preferences
- Develop novel estimator: lower variance, better confidence interval coverage
- Delivers benchmark estimates of LRSD