

Uncertainty shocks as second-moment news shocks

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Abstract

This paper provides new empirical evidence on the relationship between aggregate uncertainty and the macroeconomy. We identify uncertainty shocks using methods from the literature on news shocks, following the observation that second-moment news is a shock to uncertainty. According to a wide range of VAR specifications, shocks to uncertainty have no significant effect on the economy, even though shocks to realized stock market volatility are contractionary. In other words, realized volatility, rather than uncertainty about the future, is associated with contractions. Furthermore, investors have historically paid large premia to hedge shocks to realized volatility, but the premia associated with shocks to uncertainty have not been statistically different from zero. We argue that these facts are consistent with the predictions of a simple model in which aggregate technology shocks are negatively skewed. So volatility matters, but it is the realization of volatility, rather than uncertainty about the future, that seems to be associated with declines. In other words, uncertainty is confounded with negative shocks – increases in uncertainty happen in bad times, which drives the observed negative correlation between output and volatility.

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1 Introduction

A growing literature in macroeconomics studies the effects of news shocks on the economy. Models with rational forward-looking agents imply that pure changes in expectations about the future – news shocks – can induce a response in the aggregate economy. The existing literature has focused on *first-moment* news shocks: news about the average future path of the economy. For example, the literature on total factor productivity (TFP) and real business cycles has studied two types of TFP shocks: surprise innovations in TFP, and news about the future level of TFP that has no effects on TFP on impact. Empirically, the literature has documented important differences in how the economy responds to the two shocks (Beaudry and Portier (2006), Barsky and Sims (2011), and Barsky, Basu, and Lee (2015)).

This paper contributes to the news shock literature by extending the estimation to *second-moment* news shocks. Whereas the work described so far studies changes in the expected future growth rates, we study changes in expected future *squared* growth rates. News about the expectation of squared innovations in future growth rates represents a change in the conditional variance – that is, it is an *uncertainty shock*. Beaudry and Portier (2014) in fact suggest precisely this conceptualization of uncertainty shocks, and we use it to obtain estimates of the real effects of uncertainty shocks.

In this paper we therefore contribute to the empirical literature on the effects of uncertainty shocks by using an identification scheme proposed in the news shock literature.¹ The effects of uncertainty shocks have received substantial attention both in the literature and in the popular press. Many real-world events, like elections, referenda, and policy decisions, have large effects on uncertainty, and a natural question is whether that uncertainty affects economic activity. Our goal is to test whether uncertainty about the aggregate economy is an important driver of fluctuations in output.

The analogy to news shocks is central to our analysis. In studies of first-moment news shocks, a distinction is made between innovations in the current level of TFP – which will also usually be correlated with changes in expectations of future TFP growth – and innovations in expected future growth rates that are orthogonal to the contemporaneous TFP innovation (i.e. shocks on date t that affect $E_t TFP_{t+1}$ but have no effect on TFP_t). That is, the aim is to identify responses to pure news shocks that affect expectations of future growth rates but have no effect on TFP on impact.

In the context of second-moment news shocks, similarly, we must distinguish between current squared growth rates and news about future squared growth rates, i.e. between $(\Delta TFP_t)^2$ and $E_t [(\Delta TFP_{t+1})^2]$. For reasons discussed below, we measure second moments using aggregate stock

¹See, among others, Alexopoulos and Cohen (2009), Bachmann and Bayer (2013), Bachmann, Elstner, and Sims (2013), Bachmann and Moscarini (2012), Baker and Bloom (2013), Baker, Bloom, and Davis (2015), Basu and Bundick (2015), Bloom (2009), Born and Pfeifer (2014), Caldara et al. (2016), Fernandez-Villaverde et al. (2011), Fernandez-Villaverde et al. (2013), and Ludvigson, Ma, and Ng (2016). The theoretical literature has developed numerous mechanisms through which uncertainty about the future could affect the economy, such as precautionary saving demand among households (e.g. Basu and Bundick (2015) and wait-and-see behavior in firm investment (Bloom (2009)).

returns instead of TFP, and we extract their expectations using options on the stock market index. So our uncertainty shock is an increase in the variance of the conditional distribution of future stock prices. The analog to the first-moment impact shock is instead the surprise in the size of the squared change in stock prices – realized volatility – in the current period.

We thus identify two shocks: a realized volatility shock and an uncertainty shock. The key distinction between the two is that realized volatility – squared stock returns in period t – is not the same as uncertainty about the future, which is instead equal to the expectation of *future* squared stock returns.² Models of the effects of uncertainty, such as those with wait-and-see effects, are driven by variation in agents’ subjective distributions of future shocks, as opposed to the realization of volatility itself. The importance of that distinction is part of the basic message of this paper; to the best of our knowledge, we are the first to highlight the importance of distinguishing the two shocks when conducting empirical studies of the effects of uncertainty shocks.

In the paper, we focus on the effects of uncertainty about the aggregate stock market (S&P 500 index), rather than uncertainty about TFP. That is, in our empirical analysis we will consider the real effects of shocks to the realized variance of the aggregate stock market (the impact shock), and of shocks to the uncertainty about future stock market returns (the uncertainty shock). Our concept of uncertainty, therefore, captures the uncertainty about the aggregate value of the largest firms in the US economy. A stock-market based measure of uncertainty has several advantages over alternative measures of uncertainty. First, we expect it to reflect various types of macroeconomic uncertainty (for example, about TFP or other macroeconomic shocks), as the value of firms is affected by the underlying shocks of the economy. Second, we can measure realized and expected volatility cleanly, using high-frequency changes in stock market prices to compute realized volatility, and using option prices to measure expected volatility. Finally, measures of stock market volatility (like the VIX) have been widely used in past research on uncertainty shocks, making it easy to compare our work to the existing literature.

Technically, we use the identification scheme of Barsky, Basu, and Lee (2015), which identifies a news shock in a VAR as the rotation of the reduced-form shocks that predicts the future level of TFP (in our case, the sum of squared future stock returns) and is also orthogonal to the reduced-form innovation to the current level of TFP (in our case, orthogonal to contemporaneous squared stock returns). In order for the identification to have any power, the VAR must include data that contains information about future volatility. We therefore include measures of option-implied volatility in the VAR. Unlike in past work, though, there is no assumption here that options directly measure agents’ expectations of future volatility. Rather, our identification just requires that they contain information about expectations; they are allowed to be contaminated with noise, e.g. time-varying risk premia or measurement error.

²Distinguishing realizations and expectations is particularly important in light of the existing empirical literature, which has effectively ignored their difference. While theoretical models are purely about forward-looking uncertainty – the variance of the conditional distribution of future outcomes – the data that has been studied is frequently about *realizations* of volatility. Bloom (2009), in fact, uses realized volatility in a VAR as a proxy for forward-looking uncertainty when the VIX is unavailable.

Across a range of VAR specifications and various assumptions about the details of the identification, we find that increases in contemporaneous realized volatility are associated with declines in output, consumption, investment, and employment, consistent with the empirical findings in Bloom (2009) and Basu and Bundick (2015). More surprisingly, though, the uncertainty/second-moment news shock is estimated to have no significant effect on the real economy. In some specifications uncertainty shocks are mildly contractionary, in others they are actually expansionary, but in no case are they statistically significant. In other words, there is no evidence in the data under our identification scheme that a second-moment news shock has any negative effect on the economy. And the difference between the responses of the economy to the realized and expected volatility shocks is itself statistically significant in our benchmark specification, indicating that the failure to find news shocks to be contractionary is not simply due to low statistical power.

In addition, a forecast error variance decomposition shows that second-moment news shocks account for less than 1 percent of the variance of employment and industrial production at almost all horizons; the 97.5 percentile of the confidence interval is less than 6 percent for horizons up to a year. Our main empirical conclusion is that uncertainty shocks – as captured by our measure – do not seem to be an important source of macroeconomic fluctuations.

These results are not caused by a lack of second-moment news or a lack of power. The news shocks have statistically significant forecasting power for future stock market volatility at horizons of 6 to 10 months (which is typical for stock market volatility and similar to the length of the uncertainty shocks measured by Bloom (2009)), and we show in regressions that option-implied volatility contributes as much to variation in expectations of future volatility as lags of volatility itself do. In other words, option market investors appear to have economically meaningful information about future uncertainty that is not contained in the time series of past realized volatility. It is that information that drives our identification.

There are a number of possible explanations for the finding that realized volatility is associated with recessions but the second-moment news shock that we identify is not. In the paper, we examine several possibilities, and conclude that the empirical patterns we document are not due to measurement error for expected volatility, nor to distortions in the expectation formation of economic agents (as implied, for example, by extrapolative expectation models). Instead, the results simply indicate that shocks to expected future volatility do not appear to be contractionary.

If uncertainty shocks are not contractionary, then realized volatility cannot not affect the economy through an uncertainty channel. What then explains why realized volatility is associated with recessions? There are two theoretical possibilities: realized volatility might have negative effects through some other channel, or recessions and realized volatility might be jointly caused by a third factor. In the paper, we provide evidence in support of this final possibility: in the last section of the paper we develop a simple model in which fluctuations in economic activity are negatively skewed and stochastically volatile. Skewness in equilibrium quantities could arise because the fundamental shocks are skewed, or because symmetrical shocks are transmitted to the economy asymmetrically (perhaps because constraints, such as financial frictions, bind more tightly in bad times; Kocher-

lakota (2000)). In either case, skewness immediately generates the observed negative empirical relation between *realized* volatility and economic activity: skewness literally says that the squared value of a variable is correlated with the variable itself.

In the model, TFP growth is negatively skewed and has time-varying volatility. The skewness is induced by a time-varying probability of medium-size downward jumps in productivity.³ This specification gives a simple way of capturing skewness and stochastic volatility – we leave the deep sources of those effects to future work. We show that the variation in the conditional volatility – which maps into the second-moment news shocks – has quantitatively small real effects, while realized volatility in the model – which is driven by the downward jumps – is correlated with declines in activity. Finally, we estimate the same VAR in the model that we estimate in the data and we find highly similar results – identified uncertainty shocks have trivial effects on output, while the realized volatility shocks are contractionary, with a similar magnitude to what is observed empirically. Moreover, the identified shocks in the simulated VAR are strongly correlated with the simulated structural shocks. The identified uncertainty shock maps into the volatility shock in the model, while the realized volatility shock maps to the jumps, providing theoretical support for our identification scheme (Basu and Bundick (2015) use a similar argument in support of their identification scheme).

There are two important further pieces of evidence in favor of the skewness hypothesis. First, changes in a wide variety of measures of real activity are negatively skewed, as are stock returns. Second, when we look at the premia investors are paying to insure against uncertainty shocks and realized shocks in financial markets, we find that investors have paid large premia for insurance against high realized volatility and extreme negative stock returns (known as the variance risk premium and the option skew or put premium, respectively) in the last 30 years, whereas the premium paid for protection against increases in *expected* volatility has historically been near zero or even positive (see for example Egloff, Leippold, and Wu (2010); Ait-Sahalia et al. (2015); Dew-Becker et al. (2016)).⁴ This is consistent with uncertainty having no effects on the economy in equilibrium. We show that the model qualitatively matches both the empirical left skewness and the large premium on realized volatility compared to shocks to volatility expectations (though the magnitude of the risk premia is small compared to the data, as is common in models of the business cycle).

A final potential theoretical explanation of our results is that uncertainty matters for the economy, but not uncertainty about the aggregate stock market. There is an incredibly wide range of different measures of uncertainty, but those associated with the stock market – most prominently, the VIX (see Bloom (2009)) – are most commonly used. That is natural as standard models imply

³It is conceptually similar to consumption-based models like Barro (2006) and production-based models like Gourio (2012), but with smaller and more frequent “disasters”, consistent with the evidence in Backus, Chernov, and Martin (2011).

⁴A large literature in finance studies the pricing of realized and expected future volatility. See, among many others, Adrian and Rosenberg (2008), Bollerslev et al. (2009), Heston (1993), Ang et al. (2006), Carr and Wu (2009), Bakshi and Kapadia (2003), Egloff, Leippold, and Wu (2010), and Ait-Sahalia, Karaman, and Mancini (2013) (see Dew-Becker et al. (2016) for a review).

that stock prices summarize information about the full range of shocks that hit the economy. Our paper provides evidence on that specific measure. Other types of uncertainty might matter (such as policy or interest rate uncertainty) but they lie outside the range of theories that our identification scheme can test.

In addition to the macroeconomic studies discussed above, our work is also closely related to an important strand of research in finance. It has long been understood in the asset pricing literature that expected and realized volatility, while correlated, have important differences (e.g. Andersen, Bollerslev, and Diebold (2007)). A jump in stock prices, such as a crash or the response to a particularly bad macro data announcement, mechanically generates high realized volatility. On the other hand, news about future uncertainty, such as an approaching presidential election, increases expected volatility (Kelly, Pastor, and Veronesi (2016)). Shocks to realized and expected future volatility are correlated, but they are not as strongly correlated as one might expect – in our sample, the correlation is only 65 percent. This means that it is possible to identify in the data shocks to expectations that are orthogonal to realizations.

To summarize, then, we provide evidence from VARs, the term structure of variance risk premia, the skewness of real activity, and a structural model of the economy that suggests that output and realized volatility in the stock market are jointly caused by negatively skewed fundamentals. That is, we find that volatility matters, but it is the *realization* of volatility, rather than news about the expectation – i.e. an uncertainty shock – that is associated with future contractions.

It is important to note that our analysis is only of the effects of fluctuations in aggregate stock market uncertainty. It simply shows that uncertainty about future stock returns, after controlling for current conditions, does not have predictive power for the future path of the economy. We do not measure variation in cross-sectional uncertainty. There are obviously many dimensions along which uncertainty can vary, and we try to understand just one here.

Our work is related to a large empirical literature that studies the relationship between aggregate volatility and the macroeconomy noted above. A wide range of measures of volatility in financial markets and the real economy have been found to be countercyclical.⁵ To identify causal effects, a number of papers use VARs, often with recursive identification, to measure the effects of volatility shocks on the economy.⁶ Ludvigson, Ma, and Ng (2015), like us, distinguish between different types of uncertainty. They show that variation in uncertainty about macro variables is largely an

⁵Gilchrist, Sim, and Zakrajsek (2014) use the same fact as a starting point for an analysis of volatility, irreversible investment, and financial frictions. See Campbell et al. (2001) (equity volatility at the index, industry, and firm level is countercyclical); Storesletten, Telmer, and Yaron (2004) and Guvenen, Ozkan, and Song (2014) (household income risk is countercyclical); Eisfeldt and Rampini (2006) (dispersion in industry TFP growth rates is countercyclical); Alexopoulos and Cohen (2009) and Baker, Bloom, and Davis (2015) (news sources use uncertainty-related language countercyclically); among many others, some of which are discussed below.

⁶See Bloom (2009) and Basu and Bundick (2015), who study the VIX; and Baker, Bloom, and Davis (2015) and Alexopoulos and Cohen (2009), who study news-based measures of uncertainty. Jurado, Ludvigson, and Ng (2015) and Ludvigson, Ma, and Ng (2015) measure uncertainty based on squared forecast errors for a large panel of macroeconomic time series (using a two-sided filter to extract a latent volatility factor). Baker and Bloom (2013) use cross-country evidence to argue that there is causal and negative relationship between uncertainty and growth.

endogenous response to business cycles, whereas shocks to financial uncertainty cause recessions.⁷ Similarly, Caldara et al. (2016) use a penalty-function based identification scheme to distinguish between the effects of uncertainty and financial conditions. A key distinction between our work and those two papers is that we focus on the distinction between uncertainty expectations and realizations. Moreover, unlike most past work (Ludvigson, Ma, and Ng (2015) and Caldara et al. (2016) excepted), our identification scheme builds on the news shock literature, rather than using a more restrictive recursive setup.

The remainder of the paper is organized as follows. Section 2 describes how we identify second-moment news shocks. Section 3 describes the data, and section 4 provides evidence on the predictability of aggregate volatility and uncertainty. We present the main VAR results in section 5. Section 6 next presents some additional supporting evidence from the returns of financial derivatives. Finally, section 7.3 describes our simple model that captures the basic features of the economy described in the earlier parts of the paper, and section 8 concludes.

2 Identification

This section describes how we identify second-moment news shocks in the data. We focus on uncertainty about the future level of the stock market. The feature of the data that we want to measure is the variance of the flow of aggregate shocks that hit the economy. We thus do not aim to measure cross-sectional dispersion in shocks or even forecast uncertainty. Equity prices are useful for summarizing information about the future path of the economy.⁸

2.1 Conditional variances

Denote the log of the total return stock index as s_t . Uncertainty about the future value of the stock market relative to its value today is measured as

$$Var_t [s_{t+n}] = E_t \left[(s_{t+n} - E_t [s_{t+n}])^2 \right] \quad (1)$$

The one-period log stock return is $r_t \equiv s_t - s_{t-1}$. If returns are uncorrelated over time and time

⁷Other papers arguing that causality could run from real activity to volatility and uncertainty include Decker, D’erasmo and Boedo (2016), Berger and Vavra (2013), Ilut, Kehrig and Schneider (2015), and Kozlowski, Veldkamp, and Venkateswaran (2016).

⁸For example, in standard investment theories, stock prices are closely related to the discounted present value of the marginal product of capital (in q theory, that link is exact). Jurado, Ludvigson, and Ng (2015), on the other hand, construct a monthly measure of forecast uncertainty for a wide range of macroeconomic variables. Our goal is to measure the variance of the *common* shocks to measures of activity, rather than the total dispersion of each measure.

periods are sufficiently short that $E_t r_{t+1} \approx 0$, we have:

$$Var_t [s_{t+n}] = E_t \left[\sum_{j=1}^{n-1} r_{t+j}^2 \right] + E_t \left[\sum_{j=1}^{n-1} r_{t+j} \right]^2 \quad (2)$$

$$\approx E_t \left[\sum_{j=1}^{n-1} r_{t+j}^2 \right] \quad (3)$$

That is, when returns are serially uncorrelated (which is very nearly true empirically, especially at short horizons), the conditional variance of stock prices on some future date is equivalent to the expected total variance of returns over that same period.⁹ As the length of a time period approaches zero, the second line becomes an equality. This representation is useful because, by writing the conditional variance as an expectation, we can directly connect to the news shock literature, which studies changes in expectations.

Whereas the literature on news about TFP studies $E_t \left[\sum_{j=1}^n \Delta t f p_{t+j} \right]$ where $\Delta t f p$ is the first difference of log TFP, here we study second-moment expectations: the expectation of future squared returns ($r^2 = (\Delta s)^2$), which is simply the conditional variance of future stock prices. Second-moment news shocks – uncertainty shocks – are shifts in expected future squared returns.

In the literature on TFP news shocks, there is also the contemporaneous innovation in TFP, $t f p_t - E_{t-1} t f p_t$. The analog here is the innovation in *realized volatility*, $r_t^2 - E_{t-1} [r_t^2]$. The conditional variance of future stock prices, $Var_t [s_{t+n}]$, is equal (when returns are calculated at high frequency) to cumulative expected future realized volatility.

In the end, then, our analysis parallels the first-moment news shock literature closely. Anywhere past work talks about $\Delta t f p$, it is replaced here with $r^2 = (\Delta s)^2$, both when looking at realization shocks and at news. First-moment news shocks are about changes in the expectation of future values of $\Delta t f p$, holding constant the current innovation in $\Delta t f p$. Second-moment news shocks are changes in the expectation of future values of $(\Delta s)^2$, holding constant the current innovation in $(\Delta s)^2$ (current realized volatility).

One last minor issue is that we have data on daily stock returns, but data on real activity only at the monthly level. We therefore aggregate volatility to a monthly frequency. Specifically, we define realized volatility in month t , RV_t , as

$$RV_t \equiv \sum_{days \in t} r_i^2 \quad (4)$$

⁹In practice, we work with daily returns where the zero-mean approximation holds strongly, as documented in the literature. In the notation of continuous time models, $E_t [r_{t,t+1}^2]$ is $O(\Delta t)$, while $E_t [r_{t,t+1}]^2$ is $O(\Delta t^2)$, where Δt is the length of a time period. So as the time period gets small, the terms involving squared expected returns become negligible.

We then have

$$\text{Var}_t [s_{t+n}] \approx E_t \left[\sum_{j=1}^n RV_{t+j} \right] \quad (5)$$

Again, the approximation is only due to discreteness – if we had truly continuous data instead of sampling only at the daily level, (3) and (5) would hold exactly. Given how small average daily stock returns are (less than 0.05 percent), the approximation errors here are quantitatively irrelevant.

To summarize, then, whereas the past literature has estimated first-moment news shocks, here we aim to estimate second-moment news shocks. Instead of measuring expected and realized growth rates or returns, we measure expected and realized *squared* returns (growth rates of the market index), which correspond to the conditional variance of future stock prices and their realized volatility in the current month. We identify the effects of uncertainty shocks by studying how news about future volatility – holding current realized volatility constant – affects the real economy.

2.2 VAR identification and estimation

We now discuss how we identify second-moment news shocks using a VAR structure similar to the existing first-moment news literature.

2.2.1 VAR structure

We estimate VARs of the form

$$\begin{bmatrix} RV_t \\ Y_t \end{bmatrix} = C + F(L) \begin{bmatrix} RV_{t-1} \\ Y_{t-1} \end{bmatrix} + \varepsilon_t \quad (6)$$

where RV_t is realized volatility from (4), Y_t is a vector including measures of real activity, variables that help forecast future values of realized volatility, and other controls, C is a vector of constants, $F(L)$ is a matrix lag polynomial, and ε_t is a vector of reduced-form innovations. Our aim is to identify two structural shocks. The first is the pure innovation to RV_t , which is simply the first element of ε_t . The second is the residual innovation in uncertainty, $\text{Var}_t [s_{t+n}]$ or, equivalently, expectations of future volatility, $E_t \sum_{j=0}^{n-1} RV_{t+j} - E_{t-1} \sum_{j=0}^{n-1} RV_{t+j}$.

The reduced-form shocks, ε_t are a rotation of a vector of uncorrelated structural shocks u_t , with $\varepsilon_t = Au_t$. The VAR has an associated moving average (MA) representation,

$$\begin{bmatrix} RV_t \\ Y_t \end{bmatrix} = (I - F(1))^{-1} C + B(L) \varepsilon_t \quad (7)$$

$$\text{where } B(L) = \sum_{j=0}^{\infty} B_j L^j = (I - F(L))^{-1} \quad (8)$$

2.2.2 Identification

We assume that the first row of A is equal to $[1, 0, \dots]$, so that the first element of u_t is simply the reduced form innovation to RV_t (the first element of ε_t). Since RV_t is measured during month t , it is entirely contemporaneous or backward-looking, whereas our ultimate goal is to measure forward-looking uncertainty.

The second element of u_t , the second structural shock, is identified as the volatility news shock. Using the MA representation, second-moment news is defined as

$$E_t \sum_{j=1}^n RV_{t+j} - E_{t-1} \sum_{j=1}^n RV_{t+j} = \left(e_1 \sum_{j=1}^n B_j \right) \varepsilon_t \quad (9)$$

where $e_1 = [1, 0, \dots]$. The parameter n determines the horizon over which the news shock is calculated. Cumulative expected volatility depends on the sum of the first rows of the MA matrices up to lag n . The innovation to expectations over horizon n is then simply the linear combination of shocks represented by $e_1 \sum_{j=1}^n B_j$. As in Barsky, Basu, and Lee (BBL; 2014) and Barsky and Sims (2011), we then orthogonalize that linear combination with respect to the innovation to RV_t (i.e. the first element of ε_t) so that the impact shock to RV_t is uncorrelated with the news shock. The BBL method is only partially identified in that it identifies two of the structural shocks and leaves the remainder unspecified.

Obviously in order to identify a news shock, the vector of state variables in the VAR, Y_t , must contain information that can reveal expectations of future volatility. We therefore include in Y_t information from financial markets. First, we include $V_{6,t}$, the option-implied volatility of stock returns over the next six months. In general $V_{6,t}$ does not include all the available financial information about uncertainty, so we also include a second variable, $slope_t$, which is the first principal component of option-implied volatilities at maturities between 1 and 6 months after orthogonalizing with respect to $V_{6,t}$. Similar to past work (e.g. Egloff, Leippold, and Wu (2010) and Ait-Sahalia, Karaman, and Mancini (2013)), we find that this principal component approximately measures the slope of the volatility term structure, hence its name.

There is no assumption here that risk premia are zero or constant or that the option-implied volatility is measured without error. The only assumption that we need for identification is that some element of Y_t contains information about future values of RV . We include option-implied volatilities because we would expect them to contain such information, but they are obviously also contaminated by risk premia and potential measurement error (e.g. due to stale prices or bid/ask spreads).

While the identification scheme described above, using $e_1 \sum_{j=1}^n B_j$, is highly general in allowing any of the variables in the VAR to help forecast volatility, the generality means that it has relatively low power since it relies on accurate estimation of many coefficients. Furthermore, we will show below that none of the variables included in the VAR except for $V_{6,t}$ and $slope_t$ are actually significant predictors of future volatility (and $slope_t$ only barely). That result is not surprising. Duffee (2011),

for example, shows that in a standard class of affine term structure models, true expectations are spanned by market prices except in knife-edge cases, even under arbitrary specifications for risk premia. Intuitively, we would expect information about future volatility to appear in the volatility term structure *somehow*, even if not in a simple manner.

In light of the fact that only $V_{6,t}$ and $slope_t$ are estimated to be significant predictors of future volatility, we consider a restricted version of the estimates in which we set to zero the elements of the vector $e_1 \sum_{j=1}^n B_j$ corresponding to the variables other than RV_t , $V_{6,t}$, and $slope_t$. This zero restriction helps increase estimation power since it substantially reduces the number of coefficients in the VAR that affect identification of the news shock.

Finally, we will see that $slope_t$ itself is at best a marginally significant predictor of future volatility, and usually is not statistically significant. We therefore also consider a specification in which we set to zero the elements of the vector $e_1 \sum_{j=1}^J B_j$ corresponding to the variables other than RV_t and $V_{6,t}$. This specification involves the strongest restrictions, that only lagged RV itself and V_6 contain information about future volatility, but those assumptions appear to be a good description of the data, and obviously they help us gain statistical power.¹⁰

We report results using all three versions of the specification: the unrestricted news shock identification, with the restriction that expectations are spanned by RV_t , $V_{6,t}$, and $slope_t$, and with the restriction that further excludes $slope_t$ from the news shock.

Out structural shocks are only identified up to some normalization. Here we rescale the two shocks – the realized volatility shock and the uncertainty shock – so that they have the same effect on uncertainty. Specifically, denote the standard IRFs where the structural shocks are normalized to have unit variance as $g_{j,k,s}$, where $g_{j,k,s}$ is the response of variable j to shock k at horizon s . We report normalized IRFs of the form

$$\hat{g}_{j,k,s} \equiv \frac{g_{j,k,s}}{\sum_{m=1}^{\bar{s}} g_{1,k,m}} \quad (10)$$

The scaling factor in the denominator is the cumulative expected effect of shock k on future RV_t up to horizon $t + \bar{s}$. In this way, the IRFs we report are scaled so that they all have a unit effect on uncertainty about the level of stock prices in period $t + \bar{s}$: they contain the same amount of news about future uncertainty. In our empirical work, we set $\bar{s} = 24$ months, which is the horizon over which we examine IRFs (past work finds that volatility shocks have half-lives of 6–12 months, so 24 months represents the point at which the average shock has dissipated by 75 percent or more).

¹⁰It may be noted that the identification in this restricted case is numerically equivalent to a Cholesky factorization in which RV moves first and V_6 second. Timing is obviously not the economic restriction that generates the identification here, though. Basu and Bundick (2015) use a Cholesky factorization to identify their uncertainty shock and show that such identification is consistent with their theoretical model. We obtain a similar result with our structural model below.

3 Data

3.1 Macroeconomic data

We focus on monthly data to maximize statistical power, especially since fluctuations in both expected and realized volatility are rather short-lived. We measure real activity using the Federal Reserve’s measure of industrial production for the manufacturing sector. Employment and hours worked are measured as those of the total private non-farm economy.

3.2 Financial data

We obtain data on daily stock returns of the S&P 500 index from the CRSP database and use it to construct RV_t at the monthly frequency. We construct measures option-implied volatilities, $V_{n,t}$, using prices of S&P 500 options obtained from the Chicago Mercantile Exchange (CME), with traded maturities from one to at least six months since 1983. Our main results focus on options with six months to maturity, which is the longest maturity for which we have consistent data. Given that shocks to stock market volatility are typically short-lived, with half lives often estimated to be on the order of six to nine months (see Bloom (2009) and Drechsler and Yaron (2011)), six-month options will contain information about the dominant shocks to uncertainty.

Using results from Bakshi, Kapadia, and Madan (2003) it is straightforward to show that the variance of the index under the pricing measure Q can be written as a function of option prices,¹¹

$$V_{n,t} \equiv Var_t^Q [s_{t+n}] \tag{11}$$

$$= 2 \int_0^\infty \frac{1 - \log\left(\frac{K}{e^{rt}S_t}\right)}{B_t(n)K^2} O(K) dK - \left(e^{rt} \int_0^\infty \frac{O(K)}{B_t(n)K^2} dK \right)^2 \tag{12}$$

Note that this formula holds generally, requiring only the existence of a well-behaved pricing measure; there is no need to assume a particular specification for the returns process. $Var_t^Q [s_{t+n}]$ is calculated as an integral over option prices, where K denotes strikes, $O_t(n, K)$ is the price of an out-of-the-money option with strike K and maturity n , and $B_t(n)$ is the price at time t of a bond paying one dollar at time $t + n$. $V_{n,t}$ is equal to the option-implied variance of log stock prices n months in the future. Computing $V_{n,t}$ with real-world data requires several steps; the appendix provides a description of our calculation methods and analyzes the accuracy of the data.

Finally, in the remainder of the paper we focus on the logs of realized and option-implied volatility ($rv_t \equiv \log RV_t$, $v_{n,t} \equiv V_{n,t}$). Given the high skewness of realized volatility, the log transformation makes the results less dependent on the occasional volatility spikes. We nevertheless also show that our results are robust to performing the analysis in levels.

¹¹The pricing measure, Q , is equal to the true (or physical) pricing measure multiplied by $M_{t+1}/E_t[M_{t+1}]$, where M_{t+1} is the pricing kernel. The result for $Var_t^Q [s_{t+n}]$ is obtained from equation 3 in Bakshi, Kapadia, and Madan (2003) by first setting $H(S) = \log(S)$ to obtain $E_t^Q [\log S_{t+n}]$ and then defining $G(S) = (\log(S) - E_t^Q [\log S_{t+n}])^2$ and inserting it into equation 3 in place of H .

3.3 The time series of uncertainty and realized volatility

Figure 1 plots the history of realized volatility along with 6-month option-implied uncertainty in annualized standard deviation terms. Both realized volatility and forward-looking uncertainty vary considerably over the sample. The two most notable jumps in volatility are the financial crisis and the 1987 market crash, which both involved realized volatility above 60 annualized percentage points and rises of $V_{6,t}$ to 40 percent. At lower frequencies, the periods 1997–2003 and 2008–2012 are associated with persistently high uncertainty, while it is lower in other periods, especially the early 1980’s, early 1990’s, and mid-2000’s. There are also distinct spikes in uncertainty in the summers of 2010 and 2011, likely due to concerns about the stability of the Euro and the willingness of the United States government to continue to pay its debts.

Panel A of Table 1 reports descriptive statistics for the series in figure 1. The mean of option-implied uncertainty is substantially higher than that of realized volatility, which indicates the presence of large risk premia. Specifically, there is a negative risk premium on volatility (Coval and Shumway 2001), which causes the prices of financial claims on volatility to be biased upward compared to realized volatility.

Panel B of Table 1 reports raw correlations of the logs of realized volatility and option-implied uncertainty with measures of real economic activity – capacity utilization, the unemployment rate, and returns on the S&P 500 (correlations are similar in levels). Both measures of volatility are correlated with all three macroeconomic variables, most strongly with capacity utilization.

4 Second-moment forecasting regressions

Since identification of the second-moment news shock depends on using the variables in the VAR to forecast future realized volatility, a natural first question is which of those variables, if any, has forecasting power. Table 2 reports results of regressions of $\sum_{j=1}^6 rv_{t+j}$ on various predictors. The first column reports results from a regression on rv_t and $v_{6,t}$. Both rv_t and $v_{6,t}$ have t-statistics of approximately 4, showing not only that they are both highly statistically significant predictors of future volatility independent of each other, but also that they have very similar marginal R^2 s (since the t-statistic is a monotone function of the marginal R^2). That is, realized volatility and the option-implied expectation seem to contain equal information about future uncertainty.

This result is important for ruling out simple forecasting models where realized volatility can be forecasted purely from its own lags. However, the first column of table 2 also shows that $v_{6,t}$ is itself not a pure measure of uncertainty – if it were, it would be expected to have a coefficient of 1 and drive out all other predictors (since $v_{6,t}$, according to (11), is the option-implied volatility of stock prices six months ahead, and thus forecasts cumulative realized volatility over that period). The fact that rv_t is also significant implies that $v_{6,t}$ is partially contaminated by risk premia.

The second column of table 2 adds information from option-implied volatilities at other maturities to the regression. Instead of including $v_{j,t}$ for many j , we summarize the information content in

the term structure through a principal components analysis. The literature on the term structure of option-implied variances finds that the cross-section of market expectations is well explained by two factors, corresponding to the level and slope of the term structure. Since $v_{6,t}$ is primarily driven by the level factor, we add a slope factor to absorb the remaining variation that is independent of $v_{6,t}$. The slope factor does not add significant forecasting power and has a t-statistic of only 1.56. There is thus no evidence that option prices beyond v_6 contribute to forecasting volatility.

The third column of table 2 extends columns 1 and 2 by including the lag of rv_t in the regression. We see that $v_{6,t}$ remains significant, implying that investors receive news about future uncertainty that cannot be simply filtered from past stock market volatility (this result also holds when further lags of rv are included).

The fourth column of table 2 adds the macroeconomic variables to the regressions. None of them are individually statistically significant, nor are they jointly significant. In the fifth column, we also try adding principal components from the large set of financial and macroeconomic time series collected by Ludvigson and Ng (2007) (which would represent using a FAVAR rather than a pure VAR). None of them has statistically significant forecasting power after controlling for rv_t and $v_{6,t}$, so we exclude them from the analysis.

The R^2 s are similar across all the specifications, and always 0.45 or less. The majority of the variation in six-month realized stock market volatility is thus unpredictable, even given information available at the beginning of the period.

Based on the evidence in table 2, we focus primarily on the version of the VAR that imposes the restriction that second-moment news depends only on rv_t and $v_{6,t}$, and not the other variables, though we also report results from the less restrictive cases. To further analyze the predictive power of those two variables for future realized volatility, figure 2 plots the coefficients β_h and γ_h from the regression

$$rv_{t+h} = \alpha_h + \beta_h v_{6,t} + \gamma_h rv_t + \varepsilon_{t,h} \quad (13)$$

where α_h is a constant and $\varepsilon_{t,h}$ a residual. We estimate the same regression for varying horizons h .

Figure 2 shows the two sets of coefficients, β_h and γ_h , for different lags h . The left-hand panel shows that lagged rv forecasts future rv , with a coefficient declining with the horizon. More interestingly, though, the right panel shows that v_6 also has significant predictive power for future volatility at all horizons, even after controlling for lagged rv . The coefficients and t-statistics for rv and v_6 are similar at all horizons, indicating that they have similar marginal R^2 s. That is, the two variables have roughly the same amount of marginal predictive power in all the regressions.

If one thought that realized volatility followed a simple AR process, then the current and lagged values would yield a sufficient statistic for expectations about the future, and v_6 and *slope* would have no marginal predictive power. The results reported in table 2 and figure 2 show that option prices contain information about uncertainty above and beyond what is contained in the history of stock market volatility, and that the predictive power from v_6 in terms of marginal R^2 is in fact highly similar to that of rv .

5 Vector autoregressions

We now report our main VAR results under the three forecasting specifications that allow progressively more variables to help forecast volatility. For all the VARs that we run, we include four lags, as suggested by the Akaike information criterion for our main specification. In the main results, the vector of variables included in the VAR is $[rv_t, v_{6,t}, FFR_t, ip_t, emp_t]$, where the latter three variables are the Fed Funds rate, log industrial production, and log employment, respectively. When $slope_t$ is allowed to help forecast volatility, it is also included. The news shocks are identified based on a forecasting horizon of 24 months. The benchmark specification uses the logs of realized volatility and six-month option-implied uncertainty due to their high skewness, but we also report results using the levels themselves. We also obtain similar results to those reported after applying backward-looking filters to the macroeconomic variables to remove trends.

5.1 Benchmark results

Our primary results focus on the case that the previous section showed is most consistent with the data, which is that the only significant predictors of future volatility are rv_t and $v_{6,t}$. We focus on this case not only for its empirical plausibility given the results of the previous section, but also because it provides maximal statistical power among the three alternatives we examine. The horizon for the identification of the news shocks is 24 months, consistent with our choice of \bar{s} (we examine robustness to this choice below).

Before reporting impulse responses it is useful to examine the coefficients in the VAR. Table 3 reports the sum of the coefficients on lagged values of rv and v_6 for a range of different specifications of the VAR. The first row reports the coefficients from the regression of log employment on rv and v_6 from a VAR that includes only those three variables. The sum of the coefficients on rv is negative, while the sum of the coefficients on v_6 is actually *positive*. High levels of realized volatility forecast low employment in the future, but high levels of option-implied uncertainty actually forecast high employment. The difference has a p-value of 0.06. This basic result will appear consistently through our analysis and will drive the other results we report below.

The second row of table 3 replaces log employment with log industrial production and finds similar though statistically weaker results. The third and fourth rows report the coefficients on employment and industrial production from our main VAR that includes those two variables, rv , v_6 , and the Fed funds rate. Finally, the bottom panel of table 3 reports results from the 1988–2006 subsample that eliminates the two biggest jumps in realized volatility. In all cases, we find similar results.

We now examine impulse response functions (IRFs), which describe the full dynamic response of the variables in the economy to the two identified shocks. As discussed above, the IRFs are scaled so that the two shocks – current rv and the identified uncertainty shock – have the same cumulative effect on volatility expectations 2–24 months in the future (i.e. not counting the impact period). That is, they are scaled so as to have the same impact on uncertainty about the level of

stock prices two years in the future.

Figure 3 presents our benchmark VAR results. The figure has three columns for the responses of rv , employment, and industrial production to the shocks. The first row shows the response of the economy to the identified rv shock. It shows that a shock to realized volatility is highly transitory: the IRF falls by half within two months, and by three-fourths within five months, showing that realized volatility has a highly transitory component.

As to the real economy, those transitory increases in realized volatility are associated with statistically and economically significant declines in both employment and industrial production. So, consistent with past work, we find a significant negative relationship between *volatility* and real activity. However, this result does not allow us to conclude that an uncertainty shock is contractionary. The reason is that this first shock is a combination of an uncertainty shock (we can see from the first panel that the shock does predict future rv after impact, so it contains news about future volatility) with a shock to *current* realized volatility (which simply reflects the occurrence of a shock during month t); by observing how the economy reacts to this combination of shocks we cannot draw conclusions about how it responds to a pure uncertainty shock.

The second row of panels in figure 3 plots IRFs for the identified uncertainty shock. First, as we would expect from equation (5), the news shock forecasts high realized volatility in the future at a high level of statistical significance. That result alone is important: it says that the identified news shock does actually contain statistically significant news. That is, the market-implied conditional variance contains information about future volatility even after controlling for current and past realized volatility.¹²

Surprisingly, the second-moment news shocks are associated with no significant change in either employment or industrial production. In fact, both employment and industrial production appear to actually increase. Furthermore, the confidence bands are reasonably narrow: at almost all horizons, the point estimate for the responses of employment and industrial production to the rv shock are outside the 90-percent confidence bands for the uncertainty shock.

To further examine the difference between the IRFs, the bottom row of panels in figure 3 reports the difference in the IRFs for the uncertainty and realized volatility shocks along with confidence bands. The two shocks have the same cumulative impacts on the future path of realized volatility (by construction, due to the scaling of the IRFs). But on impact they obviously have different effects on rv on impact due to the identifying assumptions.

The two other panels in the bottom row of figure 3 plot the difference between the IRFs for industrial production and employment. We see that the difference is significant at the 5-percent level for employment and at the 10-percent level for industrial production. So innovations in rv are followed by statistically significant declines in real activity, while uncertainty shocks are not, and that difference itself is statistically significant.

¹²It may be noted that uncertainty is forecast to be high for only 6–10 months, which is a shorter horizon for the news than is often observed in studies of TFP growth news (e.g. BBL). This result is consistent with other work on uncertainty, like Bloom (2009) and Basu and Bundick (2015).

Figure 3 shows overall that under our baseline identification scheme, rv and the uncertainty shock have identical effects on future uncertainty (by construction) but markedly different effects on the economy. What can explain that difference? In terms of the identification, both shocks are normalized to have identical effects on uncertainty about the future, but the rv shock has a large effect on realized volatility *on impact*. It is this initial impact effect that seems to be associated with declines in output. The results in figure 3 therefore show that periods of high realized volatility are associated with declines in activity, but uncertainty shocks – identified as second moment news shocks – under this identification scheme and with this set of variables, have no significant effect on the economy.

5.1.1 Forecast error variance decompositions

To further understand the importance of the uncertainty and rv shocks, figure 4 reports forecast error variance decompositions. As in figure 3, we report the effect of the rv shock, the uncertainty shock, and their difference. The realized volatility shock explains 15 percent of the variance of employment and 5 percent of the variance of industrial production at most horizons, while the point estimates for the fraction of the variance accounted for by expected volatility are close to zero. The upper end of the 95-percent confidence interval for the news shock is below 5 percent for the first 10 months. The upper end of the 95-percent confidence interval for the rv shock, though, reaches as high as 25 percent for employment and 20 percent for industrial production 10 months ahead, indicating that RV can potentially be an important driver of the real economy (though this is *not* a causal statement – in fact we provide below a simple model that matches the VAR results and in which RV and output are jointly determined).

5.1.2 Quarterly data

In order to examine the effects of our two shocks on a wider range of variables, we also estimate a VAR using quarterly data similar to that of Basu and Bundick (2015) that includes, in addition to the two volatility series, GDP, consumption, investment, hours, the GDP deflator, the M2 money supply, and the Fed Funds rate (using the Wu and Xia (2014) shadow rate when the zero lower bound binds). Appendix figure A.6 shows that following an increase in realized volatility, we obtain the same comovement emphasized by Basu and Bundick (2015): output, consumption, investment, and hours worked all decline, all statistically significantly. As we would expect, investment is most sensitive to the shock to realized volatility, with a peak response four times larger than that of output and six times larger than that of consumption. For uncertainty we again find no statistically or economically significant effects, with small initial declines and subsequent rebounds. Furthermore, the magnitude of the point estimates for the declines following expected volatility shocks is again not only statistically insignificant but also far smaller than the declines in response to realized volatility. The differences in the IRFs are again themselves marginally significant, indicating that the failure to find a significant result for the news shock is not due to low power.

5.1.3 Estimation through local projections

In order to ensure that our results are not driven by the particular structure of the VAR that we examine, we estimate impulse responses using the nonparametric method of Jordà (2005). The Jordà method uses local projections that estimate impulse responses essentially as partial derivatives. They rely much less strongly on the internal propagation of a VAR. Specifically, the Jordà projection estimates the partial derivative of the expected value of a variable, $E_t Y_{t+j}$, with respect to the current values of the state variables, X_t . That is, it yields $\partial E_t Y_{t+j} / \partial X_t$, which is exactly the definition of an impulse response function in a VAR, but the Jordà method is far less reliant on the strict VAR structure. In particular, the IRF at horizon j is obtained through direct regressions of Y_{t+j} on X_t and its lags. The difference between the Jordà projection and the VAR IRF is therefore equivalent to the difference between a direct and an iterated forecast (see Marcellino, Stock, and Watson (2006)).

Figure 5 reports estimates and confidence intervals for IRFs using the Jordà (2005) projection method. The red dotted lines indicate the baseline IRFs from figure 3. Across all nine panels in the figure, the local projections, even though they have none of the internal propagation of the VAR, yield IRFs that are highly similar to what we obtained through OLS estimation of the VAR. Increases in rv are associated with declines employment and industrial production, while increases in the identified uncertainty shock are not, though in this case the difference in the IRFs has p-values closer to 0.15 than the 0.05 that we obtain in some other specifications. The results in figure 5 thus show that our results are robust to an alternative specification that does not rely on the internal propagation of the VAR.

5.1.4 Further robustness tests

We examine a range of perturbations of our main specification from figure 3. First, we consider alternative orderings of the variables in the VAR. The effects of the ordering depend ultimately on the correlation matrix of the innovations, which we report below:

	rv	v_6	Fed Funds	Empl.	IP
rv	1				
v_6	0.66	1			
Fed Funds	-0.02	-0.07	1		
Empl.	0.04	0.10	0.03	1	
IP	-0.01	0.10	0.03	0.55	1

Consistent with standard models of equity volatility (e.g. ARCH and GARCH models), the shocks to rv and v_6 are correlated, but far from collinear. However, their innovations are almost completely uncorrelated with those in the other variables, implying that if the news shock were orthogonalized not just to the shock to rv_t but to all the macro variables also, its IRF would remain unchanged (which we have confirmed in results not reported here).

Appendix figure A.5 reports results from a monthly VAR analogous to that of figure 3 where we reverse the ordering of the realized volatility and news shocks. That is, the main results involve orthogonalizing the news shock with respect to the innovation to rv , so that it has no effect on contemporaneous realized volatility by construction. In the alternative ordering in figure A.5, the first shock is the entire revision in second moment expectations ($e_1 \sum_{j=1}^n B_j$), which is correlated with rv , while the second shock is the residual innovation in rv . That is, it is an innovation in rv that has no effect on expected future volatility ($Var_t[s_{t+n}]$), in some sense a purely transitory shock to realized volatility.

The figure shows that both shocks have essentially the same effect on employment and industrial production. That result is entirely consistent with our main analysis. Recall that in the main analysis, the identified rv and uncertainty shocks have similar effects on future volatility, but they are identified so that only the rv shock affects rv on impact. In figure A.5, on the other hand, the shocks have very *similar* effects on rv on impact, and they are now distinguished by having *different* effects on future volatility.

What figures 3 and A.5 show together is that when two shocks have the same initial effect on rv , they have the same effects on output (figure A.5) whereas when they have different effects on rv , they have different effects on output (figure 3). In other words, figures 3 and A.5 together show that it is the impact of a shock on contemporaneous realized volatility, not its effect on uncertainty, that determines how it affects output.

Figures A.7 and A.8 in the appendix report a range of additional robustness tests. Figure A.7 shows the response of log employment to the rv shock and the uncertainty shock, and figure A.8 reports the response of log industrial production. In each figure, the rows correspond to a different specifications of the model. The left panels report responses to shocks to rv , while the middle panels report the responses to uncertainty shocks. The right panel reports the difference between the two impulse-response functions. The figures examine four robustness tests:

1. Detrending by a one-sided HP filter
2. Limiting the sample to 1988–2006 to remove the jumps in volatility in 1987 and 2008.
3. Controlling for the level of the S&P 500 in the VAR before the identified shocks.
4. Using RV and V_6 (i.e, in levels rather than logs).

The results of the robustness tests are qualitatively and quantitatively consistent with our baseline results.

Finally, we also find similar results in unreported additional robustness tests: using three-month expectations v_3 ; using the SVIX of Martin (2015), as an alternative measure of option-implied uncertainty; using different detrending parameters when HP-filtering the series; using variance swap data instead of option data to construct v_6 (for the period 1996-2013); using the 12-month variance swap to construct v_{12} (for the period 1996-2013); using different subsamples of data, including focusing only on the most recent period (starting in 1990).

5.2 Allowing more variables to predict second moments

We now examine results that, first, allow $slope_t$ to also forecast future volatility, and, second, allow all the variables in the VAR to help forecast volatility. The basic summary of the results in this section is that the findings are qualitatively similar to what is reported above, but substantially weaker statistically since they require estimating more coefficients in constructing the news shock. That is, the model is now forced to estimate the response of future volatility to all the variables in the VAR, even though we already know from table 2 that they are not relevant predictors. We include these results because they are most general and impose the smallest number of restrictions.

Figure 6 reports IRFs for the shock to rv and the news shock identified under the assumption that $Var_t[s_{t+n}]$ is spanned by rv_t , $v_{6,t}$, and $slope_t$. Since the identification of the rv shock is unchanged, its IRFs are nearly identical to those in figure 3 – shocks to realizations of volatility have statistically significant negative effects on employment and industrial production (the only difference comes from the fact that now the slope factor is included in the VAR). Furthermore, with this alternative identification of the news shock there is also again no evidence that the news shocks have any significant effect on real activity, though in this case the confidence bands are much wider than in the benchmark results, due to the weaker restrictions. The bottom row of figure 6 shows that the response of real activity to the rv shock is again more negative than the response to the news shock, but that difference is now not statistically significant.

Figure 7 reports associated variance decompositions. Consistent with the previous results, the uncertainty shock is still estimated to account for a minimal fraction of the variance of real activity, but again the confidence bands are much wider than in the benchmark specification.

Finally, figures 8 and 9 report results using the completely unrestricted identification scheme from BBL for the news shock. In this case, all variables in the VAR can drive uncertainty. Similar to figure 6, we again find no evidence that the news shock has negative effects on output, but again the confidence bands are wide. The difference between the IRFs for rv and news is again statistically insignificant.

The appendix replicates the analysis from this section using alternative values of the forecast horizon used for identification of the news shocks (the benchmark results are invariant to the choice of horizon). In none of the specifications is the response of employment or industrial production to the news shock statistically significant in either direction, and the difference between the responses to the news and rv shocks is also itself never significant.

This section shows that when we identify the news shock using weaker assumptions than in our main analysis (again recalling that the identifying assumptions for the main analysis are supported by the forecasting regressions in section 4), uncertainty shocks continue to have no significant effect on real activity, either in terms of its IRFs or in terms of forecast error variance decompositions. The weaker identifying assumptions in this section reduce our statistical power sufficiently, though, that it is harder to claim that we have a well-identified zero for the effect of uncertainty shocks, especially in the fully unrestricted identification scheme.

6 Evidence from the price of insurance against volatility shocks

The results above use data from stock options to provide information about second-moment expectations. If risk premia were zero or constant, option-implied volatility would be an incredibly valuable data source since it would give a direct measure of volatility expectations. That is not the case, though, which is why we are forced to use the news shock identification scheme. So in the VARs, risk premia are a contaminant that make identification more difficult.

But risk premia actually contain information that can be useful to complement and support the previous macroeconomic analysis. Risk premia reveal how much investors are willing to pay to hedge against certain risks. By looking at risk premia of volatility-related securities (variance swaps and portfolios of options) we now show that investors have historically paid large premia for insurance against increases in realized volatility, but not for insurance against increases in market-implied uncertainty. That suggests that investors do not view periods in which uncertainty rises as having high marginal utility (i.e. as being bad times).

A one-month variance swap is an asset whose final payoff is the sum of daily squared log returns of the underlying index (the S&P 500, in our case) over the next month. That asset gives the buyer protection against a surprise in equity return volatility (rv) over the next month. If investors are averse to periods of high realized volatility, then, we would expect to see negative average returns on one-month variance swaps, reflecting the cost of buying that insurance. A simple way to see that is to note that in general the Sharpe ratio of an asset, the ratio of its expected excess return to its standard deviation, is

$$\frac{E_t[R_{t+1} - R_{f,t+1}]}{SD_t[R_{t+1}]} = -corr_t(R_{t+1}, MU_{t+1}) \quad (14)$$

for any return R_{t+1} , where $R_{f,t+1}$ is the risk-free rate and MU_{t+1} denotes the marginal utility of consumption on date $t + 1$. Assets that covary positively with marginal utility, and hence are hedges, earn negative average returns. So if realized volatility is high in high marginal utility states (in most models, bad times), then one-month variance swaps will earn high Sharpe ratios.

The first point on the left in the left-hand panel of Figure 10 plots average annualized Sharpe ratios on 1-month S&P 500 variance swaps between 1996 and 2014.¹³ The average Sharpe ratio is -1.4, approximately three times larger (with the opposite sign) than the Sharpe ratio on the aggregate equity market. In other words, investors have been willing to pay extremely large premia for protection against periods of high realized volatility, suggesting that they view those times as particularly bad (or as having very high marginal utility).

Now consider a j -month variance forward, whose payoff, instead of being the sum of squared returns over the next month ($t + 1$), is the sum of squared returns in month $t + j$ (so then the one-month variance swap above can also be called a 1-month variance forward). If an investor

¹³The data is described in Dew-Becker et al. (2016); it is obtained from a large asset manager and Markit, but may be closely approximated by portfolios of options, for which prices are widely available (e.g. from Optionmetrics).

buys a j -month variance forward and holds it for a single month, selling it in month $t + 1$, then the variance forward protects them over that period against *news* about volatility in month $t + j$. That is, if we get news that volatility will be higher in the future, it will affect the holding period return on that j -period forward. The left-hand panel of figure 10 also plots one-month holding period Sharpe ratios for variance forwards with maturities from 2 to 12 months. We see that for all maturities higher than 2 months, the Sharpe ratios are near zero, and in fact the sample point estimates are positive. The Sharpe ratios are also all statistically significantly closer to zero than the Sharpe ratio on the one-month variance swap.

The left panel of Figure 10 therefore shows that there is something special about the surprise in realized volatility compared to news about volatility going forward. Investors are willing to pay large premia for protection against surprises in realized volatility, but news about future uncertainty has a much smaller – or even zero – premium. Realized volatility thus appears to have a large positive correlation with marginal utility, while shocks to expected volatility have a correlation that is much closer to zero.

Using the options data described above, it is possible to extend those results further, back to 1983. The right-hand panel of figure 10 reports the average shape of the term structure of variance forward prices constructed from data on S&P 500 options (we study the term structure with this data because it is estimated more accurately than returns). The variance forwards are constructed from synthetic variance swaps, a calculation almost identical to our calculation of $Var_t^Q [s_{t+n}]$. The term structure reported here is directly informative about risk premia. The average return on an n -month variance claim is:

$$E \left[\underbrace{\frac{F_{n-1,t} - F_{n,t-1}}{F_{n,t-1}}}_{\text{Average return}} \right] \approx \underbrace{\frac{E[F_{n-1}] - E[F_n]}{E[F_n]}}_{\text{Average slope}} \quad (15)$$

where $F_{n,t}$ is the price on date t to a claim to realized stock market volatility in month $t + n$. The slope of the average term structure thus indicates the average risk premium on news about volatility n months forward. If the term structure is upward sloping, then the prices of the variance claims fall on average as their maturities approach, indicating that they have negative average returns. If it slopes down, then average returns are positive.

The right-hand panel of Figure 10 plots the average term structure of variance forward prices for the period 1983–2013. The term structure is strongly upward sloping for the first two months, again indicating that investors have paid large premia for assets that are exposed to realized variance and expected variance one month in the future. But the curve quickly flattens, indicating that the risk premia for exposure to fluctuations in expected variance further in the future have been much smaller.

The asset return data says that investors appear to have been highly averse to news about high realized volatility, while shocks to expected volatility do not seem to have been related to

marginal utility. The confidence intervals that we obtain are sufficiently wide that we cannot claim that shocks to expected future volatility do not earn an economically meaningful negative risk premium. What we can say, though, is that investors seem to have cared over our sample *much more* about surprises in realized volatility than in uncertainty.

Figure 10 then confirms and complements the results from our VAR, that show that shocks to rv are associated with recessions but uncertainty shocks are not. As revealed by figure 10, investors think that realized volatility is associated with downturns, while news about expected future volatility (i.e. an uncertainty shock) is not. That is, realized volatility appears to be much more strongly correlated with marginal utility than uncertainty.

7 Interpretation of the empirical results

The paper thus far has provided empirical evidence on two basic points. First, surprises in realized volatility in the stock market are associated with future declines in real activity, while uncertainty shocks, identified as second-moment news, are not. Second, investors have historically paid large premia to hedge shocks to realized volatility, but have paid premia that have averaged to nearly zero to hedge shocks to uncertainty.

There are a number of possible explanations for these findings. In this section, we consider first various explanations for why aggregate uncertainty shocks may not have real effects in our data, and we specifically rule out several alternative explanations. Then we explore what model of the economy is consistent with the irrelevance of uncertainty shocks but at the same is consistent with realized volatility being associated with recessions.

7.1 Why uncertainty shocks are not recessionary

A natural interpretation of our results is that uncertainty shocks simply do not have significant negative effects on the economy. This could be due to several factors: because uncertainty may have differential effects across the economy and the aggregate effect is small; because fiscal and monetary policy can counteract uncertainty shocks; or because uncertainty might even be good for some parts of the economy (for example, the Oi-Hartman-Abel effect suggests volatility can be beneficial to producers).

But there are other possible reasons why we may find in our data that uncertainty shocks do not induce recessions. A first possibility is that the uncertainty that we identify from the VARs may not capture agents' actual expectations of future volatility; then the fact that our second-moment news shock has no real effect on the economy may simply reflect a mismeasurement in expected volatility. At the same time, realized volatility – which does seem to affect the economy – could appear to matter in our data because it captures future expected volatility with less noise.

Our empirical results allow us to directly reject that interpretation. The results in table 2 and figures 2 and 3 all show that the option-based measure of expected volatility (v_6) has as much

predictive power for future realized volatility as lagged realized volatility itself. In fact, as table 2 shows, option-implied volatility is the only variable among the several macro/finance variables considered that actually has any forecasting power for future realized volatility, conditional on lagged realized volatility. So we see that investors clearly have expectations for future volatility, as reflected in option prices, that reflect more than simply current realized volatility.

Closely related is the possibility that people have extrapolative expectations about future realized volatility, so their uncertainty is fully summarized by current realized volatility. If this was the case, then implied volatility from options (which directly depends on people's expectations of future volatility) should itself depend on lagged realized volatility, and should not contain any additional information for predicting future realized volatility. But as discussed above, in the data we find that option prices contain important additional information about future realized volatility, suggesting that extrapolative expectations are not the reason why uncertainty shocks seem to have no effect on the macroeconomy.¹⁴

Another way to evaluate how well our measure of uncertainty captures expectations of future volatility is to compare it with alternative measures of uncertainty that are not based on option prices. Table 6 reports predictive regressions of future 6-month realized volatility (as in table 2) using not option-implied uncertainty, as in our baseline analysis, but also two additional uncertainty measures: the Baker, Bloom and Davis (2016) measure, based on newspaper textual analysis, and a measure of forward-looking uncertainty constructed from the Michigan Consumer Survey.¹⁵ The table shows that neither of these measures actually contains any information useful in predicting future volatility beyond what is already contained in lagged realized volatility; these two measures, therefore, would not be useful for identifying second-moment news shocks (at least for future stock returns). On the other hand, option prices continue to have forecasting power when controlling for lagged volatility as well as these two alternative measures of uncertainty. This evidence corroborates the argument that option prices contain information about expectations of future volatility, allowing us to identify second-moment news shocks. To be clear, though, as discussed above, there is no assumption that option-implied volatility is literally equal to agents' expectations; rather, options simply contain useful information about expectations. The VAR identification scheme is what actually measures the news shocks.

To summarize, the evidence suggests that changes in uncertainty about the future do not actually affect the real economy, and this is due neither to measurement problems nor to distortions

¹⁴Another related possibility is that the people who price options have uncertainty that differs from those who make economic decisions. So, it is in theory possible that the people that determine macroeconomic outcomes have extrapolative expectations and only base their decision on lagged realized volatility, whereas stock and option investors are rational and trade based on additional information about future volatility. This explanation would require an extraordinary degree of disconnect between the real economy, the stock market, and the option market. Given that the options market is a trillion-dollar market, known to be well integrated with the rest of financial markets, and available to essentially all investors, an explanation based on such segmentation seems unlikely. This tight integration is in fact the very reason why existing studies of uncertainty shocks like Bloom (2009) have used option-implied volatility (the VIX).

¹⁵More precisely, the measure corresponds to the fraction of respondents in the survey who say they plan on delaying car purchases due to economic uncertainty.

in expectation-formation mechanisms of investors. In addition, it is worth remembering that the fact that uncertainty does not affect the economy is also directly consistent with the pricing results in section 6.

7.2 Explaining the real effects of realized volatility *and* uncertainty shocks

We now consider what theories can explain *both* why realized variance is associated with recessions, and why uncertainty shocks do not appear to be recessionary. Since our identification method, just like in the standard news-shock literature, focuses on estimating the effects of the news shock (uncertainty shock), we do not take a stand on the causal determinants of the impact shock (realized volatility). It could be that realized volatility might have direct negative effects on economic activity, or recessions and realized volatility might be jointly caused by a third factor.

While we cannot fully determine the causal determinants of the relation between realized volatility and the macroeconomy, we provide support in this section for the hypothesis that realized volatility and recessions are due to a third factor – namely, a left-skewed shock that affects the economy.

In particular, in the next subsection we develop a simple model in which fluctuations in economic activity are negatively skewed and stochastically volatile. Skewness in equilibrium quantities could arise because the fundamental shocks are skewed, or because symmetrical shocks are transmitted to the economy asymmetrically (perhaps because constraints, such as financial frictions, bind more tightly in bad times; Kocherlakota (2000)). In either case, skewness immediately generates the observed negative empirical relation between *realized* volatility and economic activity: skewness literally says that the squared value of a variable is correlated with the variable itself.

Not only the model will generate a correlation between realized volatility and recessions; it will also provide reduced-form empirical evidence in favor of the skewness hypothesis, and it will match the other patterns we document: that uncertainty is not contractionary, and that realized volatility, but not expected volatility, is priced in asset markets (see section 6).

7.3 Equilibrium model and further evidence

We build a simple stylized structural model extending the classic RBC model. Dew-Becker et al. (2016) argue that the asset return data described above is consistent with a model with periodic downward jumps in real output. Intuitively, when output (or technology) growth is skewed to the left, large shocks, which are associated with high realized volatility, tend to be negative. That is simply the definition of left skewness: the squared innovation is negatively correlated with the level of the innovation.

We first discuss evidence that there is left skewness in economic activity and then describe the model and show that estimated impulse responses to uncertainty and realized volatility shocks in the model match what we have found in the data.

7.3.1 Skewness

A potential source of negative correlation between output and realized volatility is negatively skewed shocks. Specifically, if some shock ε is negatively skewed, then $E[\varepsilon^3] < 0 \Rightarrow cov(\varepsilon, \varepsilon^2) < 0$. That is, negative skewness implies a negative correlation between ε^2 and ε itself. So high realized volatility (ε^2) should be associated with downturns. The obvious question, then, is whether shocks to output and asset returns are actually skewed to the left. There are large literatures studying skewness in both aggregate stock returns and economic growth. We therefore provide just a brief overview of the literature and the basic evidence.

Table 4 reports the skewness of monthly and quarterly changes in a range of measures of economic activity. Nearly all the variables that we examine are negatively skewed, at both the monthly and quarterly levels. One major exception is monthly growth in industrial production, but that result appears to be due to some large fluctuations in the 1950s. When the sample is cut off at 1960, the results for industrial production are consistent with those for other variables.

In addition to real variables, table 4 also reports realized and option-implied skewness for S&P 500 returns.¹⁶ The implied and realized skewness of monthly stock returns is substantially negative, and in fact surprisingly similar to the skewness of capacity utilization. The realized skewness of stock returns is less negative than option-implied skewness, which is consistent with investors demanding a risk premium on assets that have negative returns in periods when realized skewness is especially negative (i.e. that covary positively with skewness).

In addition to the basic evidence reported here, there is a large literature providing much more sophisticated analyses of asymmetries in the distributions of output and stock returns. Morley and Piger (2012) provide an extensive analysis of asymmetries in the business cycle and review the large literature. They estimate a wide range of models, including symmetrical ARMA specifications, regime-switching models, and frameworks that allow nonlinearity. The models that fit aggregate output best have explicit non-linearity and negative skewness. Even after averaging across models using a measure of posterior probability, which puts substantial weight on purely symmetrical models, Morley and Piger find that their measure of the business cycle is substantially skewed to the left, consistent with the results reported in table 4. More recently, Salgado, Guvenen, and Bloom (2016) provide evidence that left skewness is a robust feature of business cycles, at both the macro and micro levels and across many countries.

The finance literature has also long recognized that there is skewness in aggregate equity returns and in option-implied return distributions (see Campbell and Hentschel (1992), Ait-Sahalia and Lo (1998), and Bakshi, Kapadia, and Madan (2003), for recent analyses and reviews). The skewness that we measure here appears to be pervasive and has existed in returns reaching back even to the 19th century (Campbell and Hentschel (1992)).

Taken as a whole, then, across a range of data sources and estimation methods, there is a

¹⁶We obtain option-implied skewness from the CBOE's time series of its SKEW index, which is defined as $SKEW = 100 - 10 \times Skew(R)$. We thus report $10 - SKEW/10$.

substantial body of evidence that fluctuations in the economy are negatively skewed. In a world of negative skewness, it is not surprising that measures of realized volatility are correlated with declines in activity, simply because skewness is related to the third moment: $E[\varepsilon^3] = E[\varepsilon \cdot \varepsilon^2]$.

7.3.2 An equilibrium model

The empirical evidence presented thus far is consistent with the view that shocks to aggregate realized volatility are associated with significant declines in macroeconomic activity while shocks to expected aggregate volatility are not. In this section we present a stylized equilibrium model that is both consistent with our evidence and close to the workhorse RBC model. We deliberately keep the model simple in order to highlight the economic channels that are at work; the model is not rich enough to provide a tight quantitative fit to the economy. But despite its simplicity, the model is qualitatively consistent with a wide variety of real and asset pricing facts, and illustrates what features a model can have to match the volatility patterns we document.

Our model is an RBC model where aggregate TFP growth is heteroskedastic and skewed to the left. We want it to be consistent with the three facts presented in this paper: (1) shocks to realized volatility are associated with declines in real activity, while shocks to expected volatility are not; (2) Sharpe ratios on short-term claims to volatility are much more negative than those on longer-term claims; and (3) output growth and equity returns are negatively skewed.

7.3.3 Model structure

Firms produce output with technology, A_t , capital, K_t , and labor, N_t ,

$$Y_t = A_t^{1-\alpha} K_{t-1}^\alpha N_t^{1-\alpha} \quad (16)$$

We set $\alpha = 0.33$, consistent with capital's share of income. Capital is produced subject to adjustment costs according to the production function

$$K_t = (1 - \delta) K_{t-1} + K_{t-1} \left(\frac{I_t}{K_{t-1}^{agg}} - \frac{\zeta}{2} \left(\frac{I_t}{K_{t-1}^{agg}} - \overline{I/K} \right)^2 \right) \quad (17)$$

where I_t is gross investment, K_t^{agg} is the aggregate capital stock (which is external to individual firm decisions), ζ is a parameter determining the magnitude of adjustment costs and $\overline{I/K}$ is the steady-state investment/capital ratio. We set $\delta = 0.08/12$ (corresponding to a monthly calibration) and $\zeta = 0.5$.¹⁷ Given the structure for adjustment costs and production, the equilibrium price and

¹⁷See, e.g., Cummins, Hassett, and Hubbard (1994) for estimates of adjustment costs similar to this value. Jermann (1998) uses similar values. $\zeta = 0.5$ is on the lower end of estimates based on aggregate data and more consistent with micro evidence, but our results are not sensitive to the choice of this parameter.

return on a unit of installed capital are

$$P_{K,t} = \frac{1}{1 - \zeta \left(\frac{I_t/K_{t-1}}{I/K} - 1 \right)} \quad (18)$$

$$R_{K,t} = \frac{\alpha A_t^{1-\alpha} K_{t-1}^{\alpha-1} N_t^{1-\alpha} + (1 - \delta) P_{K,t}}{P_{K,t-1}} \quad (19)$$

We assume there is a representative agent who maximizes expected utility,

$$\max_{C,N} \sum_{j=0}^{\infty} \beta^j \left(\log \left(C_{t+j} - bC_{t+j-1}^{agg} \right) - \theta \frac{N_{t+j}^{1+\gamma}}{1+\gamma} \right) \quad (20)$$

subject to the budget constraint

$$C_t + I_t \leq Y_t \quad (21)$$

Agents have log utility over consumption minus an external habit. We set the magnitude of the habit to $b = 0.8$ to help generate smoothness in consumption. β is set to $0.99^{1/12}$, γ to $1/3$ for a Frisch elasticity of 3, and θ to generate steady-state employment of $1/3$.

The model is closed by the Euler equation and the optimization condition for labor,

$$1 = E_t \left[\beta \left(\frac{C_{t+1} - bC_t^{agg}}{C_t - bC_{t-1}^{agg}} \right)^{-\rho} R_{K,t+1} \right] \quad (22)$$

$$\frac{\theta N_t^\gamma}{(C_t - bC_{t-1}^{agg})^{-\rho}} = (1 - \alpha) A_t^{-\alpha} K_{t-1}^\alpha N_t^{-\alpha} \quad (23)$$

We model realized volatility, as in the empirical analysis as the squared excess return on capital,

$$RV_t = (R_{K,t+1} - R_{f,t})^2 \quad (24)$$

where $R_{f,t}$ is the risk-free rate, with $1 = E_t \left[\beta \left(\frac{C_{t+1} - bC_t^{agg}}{C_t - bC_{t-1}^{agg}} \right)^{-\rho} R_{f,t} \right]$. It is then straightforward to construct prices on claims to future realized volatility. The price of a claim to RV_{t+j} on date t is denoted $P_{V,j,t}$.

The only exogenous variable in the model is technology, A_t , which follows the process

$$\Delta \log A_t = \sigma_{t-1} \bar{\sigma}_\varepsilon \varepsilon_t - J(\nu_t - \bar{p}\sigma_{t-1}) + \mu \quad (25)$$

$$\log \sigma_t = \phi_\sigma \log \sigma_{t-1} + \sigma_\sigma \eta_t + \kappa_{\sigma,A} \Delta \log A_t \quad (26)$$

$$\varepsilon_t, \eta_t \sim N(0, 1) \quad (27)$$

$$\nu_t \sim \text{Bernoulli}(\bar{p}\sigma_{t-1}) \quad (28)$$

Technology follows a random walk in logs with drift μ , set to 2 percent per year. ε_t is a normally distributed innovation that affects technology in each period, while ν_t is a shock that is equal to

zero in most periods but equal to 1 with probability $\bar{p}\sigma_{t-1}$ – that is, it induces downward jumps in technology, with J determining the size of the jump and \bar{p} the average frequency. σ_t determines the volatility of shocks to technology. It is itself driven by two shocks: an independent shock η_t and also the innovations to technology in period t . A positive technology shock may feed into lower volatility in the future. The volatility process thus has two features that will be important in matching the data: it has news shocks, and it is countercyclical for $\kappa_{\sigma,A} < 0$.

ϕ_σ and σ_σ are calibrated so that $\log \sigma_t$ has a standard deviation of 0.35 and a one-month autocorrelation of 0.91, consistent with the behavior of the VIX. $\kappa_{\sigma,A}$ is set to -4.37, which implies that a jump in technology, $J\nu_t$, increases σ_t by one standard deviation, generating countercyclical volatility. $\bar{\sigma}_\varepsilon$ is set so that normally distributed shocks on average generate a standard deviation of output growth close to the value of 1.92 we observe empirically. Jumps on average reduce technology by 8 percent (which is 3.2 times $\bar{\sigma}_\varepsilon$, the average standard deviation of the Gaussian TFP shocks) and are calibrated to occur once every 10 years on average. We thus think of them as representing small disasters or relatively large recessions (consistent Backus, Chernov, and Martin (2011) and with the view of skewed recessions in Salgado, Guvenen, and Bloom (2016)), rather than depression-type disasters.¹⁸

We solve the model by projecting the decision rule for consumption on a set of Chebyshev polynomials up to the 8th order (a so-called global solution) to ensure accuracy not only for real dynamics but also for asset prices and realized volatility. Integrals are calculated using Gaussian quadrature with 20 points. Euler equation errors are less than $10^{-4.3}$ across the range of the state space that the simulation explores and have an average absolute value of $10^{-4.8}$. The use of a global solution method allows for high accuracy in the solution, but also makes it infeasible to search over many parameters or estimate the model, which is why we explore just a single calibration here.

7.3.4 Simulation results

We examine three sets of implications of the model: VAR estimates, risk premia, and skewness. All results are population statistics calculated from a simulation lasting 10,000 years.

Table 5 reports basic moments of returns on capital and growth rates of output, consumption, and investment. The model generates negative skewness in all four variables in the table, consistent with the data, but the skewness is much larger than is observed empirically. Mean growth rates of real variables are similar to the data, though mean stock returns are much smaller (that is a common failure in real business cycle models unless risk aversion is raised to extremely high levels as in Tallarini (2000)). The standard deviation of output is almost identical to the data, while consumption is less and investment more volatile; the gap between the two is smaller than observed empirically, however. Table 5 thus suggests that the model generates moments that are broadly consistent with the data, in particular generating comovement among aggregate variables

¹⁸A realistic extension of the model would be to allow for jumps to be drawn from a distribution, rather than all having the same size. See, e.g., Barro and Jin (2011).

(the three growth rate series in table 5 have correlations between 0.53 and 0.88) and volatilities that are empirically reasonable.

Figure 11 plots the Sharpe ratios of volatility claims in the model that correspond to the forward volatility claims examined in Figure 10. As in Figure 10, the Sharpe ratio of the one-month asset, which is a claim to realized volatility, is far more negative than the Sharpe ratios for the claims with longer maturities. Intuitively, this is because shocks to volatility expectations, η_t , have relatively small effects on consumption, hence earning a small risk premium. Shocks to realized volatility, on the other hand, tend to isolate the jumps, ν_t , (as we will show below), so they earn larger premia. The magnitudes are far smaller than observed in the data. This could be rectified through the use of Epstein–Zin preferences, but that would make obtaining a global numerical solution of the model far more difficult.

Finally, the solid lines in figure 12 summarize the results of VARs estimated from simulations of the model. The VAR in the simulations replicates the one used in the main analysis above. In particular, it includes realized volatility, option-implied expected volatility ($P_{V,1,t}$), and the level of output, using the same news shock identification as above.

We see, as in the main results, that a unit standard deviation shock to realized volatility has a highly transitory effect on realized volatility and a negative effect on output (of a similar magnitude to what is observed empirically). The second-moment news shock has a predictive effect on future realized volatility, but only a quantitatively trivial effect on output. The bottom row shows the difference in the IRFs, and we see that the RV shock has substantially more negative effects on output than the uncertainty shock.

The VAR results are notable because they replicate the results observed empirically even though there is no structural “realized volatility shock”. Rather, the identified RV shock essentially is the jump in TFP in the model ($J\nu_t$). To see that, we report the correlations between the VAR-identified shocks and the structural shocks in the model in the bottom section of table 5.

The RV shock is correlated nearly exclusively with $J\nu_t$, the jump shock in the model. So the VAR successfully identifies the jumps as realized volatility shocks, which are then structurally, but obviously not causally, related to declines in output. The identified uncertainty shock, as we would hope, is, similarly, almost purely correlated with η_t , the volatility news shock. Finally, the third shock from the VAR – which is simply a residual unexplained by the *RV* and uncertainty shocks – is primarily correlated with ε_t , the small shock to technology. So our main VAR specification does a good job in this setting – a non-linear production model – of actually identifying true structural shocks and also fitting the qualitative behavior of our empirical VAR analysis.

The fact that the shocks identified by the VAR are very similar to the structural shocks in the model suggests that the impulse responses estimated by the VAR should be very similar to the effects of the shocks in the structural model itself. Figure 12 therefore plots, in addition to the IRFs estimated from the simulation, the responses of realized volatility, expected volatility, and output, to shocks to $J\nu_t$ and η_t – the structural jump shock and volatility news shock, respectively. The structural IRFs are the dashed lines.

We see that the response of realized and expected volatility to the VAR-estimated RV and uncertainty shocks are nearly identical to the responses to the true structural shocks $J\nu_t$ and η_t (recall that all shocks are scaled to have the same effect on expected future volatility, which is why they match closely in the middle panel on the left side of the figure). Most importantly, the response of output to the estimated shocks is rather similar to the response to the structural shocks. Output falls by 0.4 percent following the estimated RV shock, while it falls by 0.6 percent following the structural shock. The second row shows that there is essentially zero response to both the estimated uncertainty shock and to η_t , even though both do increase uncertainty and future realized volatility.

This section thus shows that a simple production model can match the basic features of the data that we have estimated in this paper: output responds negatively to shocks to realized volatility but not to shocks to uncertainty, there is a much larger risk premium for realized than expected volatility, and economic activity and stock returns are both skewed to the left.

8 Conclusion

The goal of this paper is to understand whether shocks to aggregate uncertainty have negative effects on the economy. We identify uncertainty shocks as second-moment news shocks and find that they are not followed by meaningful declines in real activity after controlling for contemporaneous volatility. The evidence we present favors the view that bad times are volatile times, not that uncertainty causes bad times. A leading hypothesized explanation for the slow recovery from the 2008 financial crisis has been that uncertainty about the aggregate economy (e.g. due to policy uncertainty) since then has been high. Our evidence suggests that aggregate uncertainty may not have been the driving force, and that economists should search elsewhere for an explanation to the slow recovery puzzle.

More constructively, this paper aims to lay out a specific view of the joint behavior of stock market volatility and the real economy. There appear to be negative shocks to the stock market that occur at business cycle frequencies, are associated with high realized volatility and declines in output, and are priced strongly by investors. The simple idea that fundamentals are skewed left can explain our VAR evidence, the pricing of volatility risk, and the negative unconditional correlation between economic activity and volatility.

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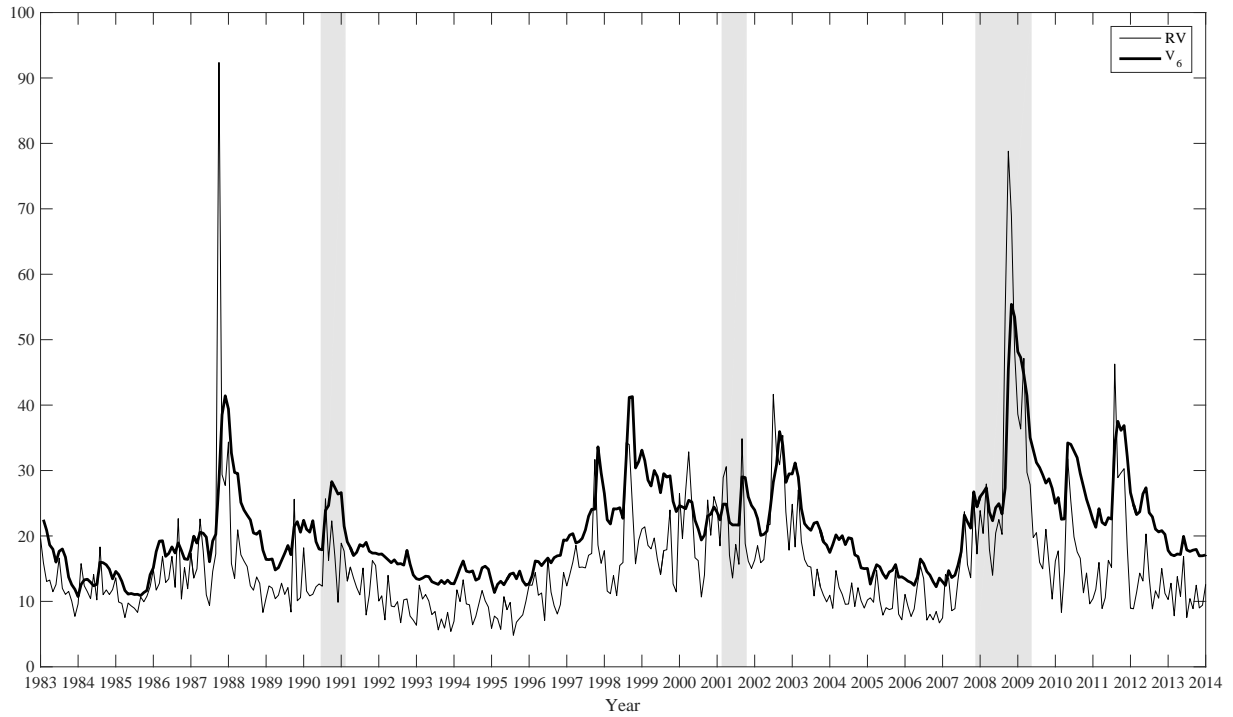
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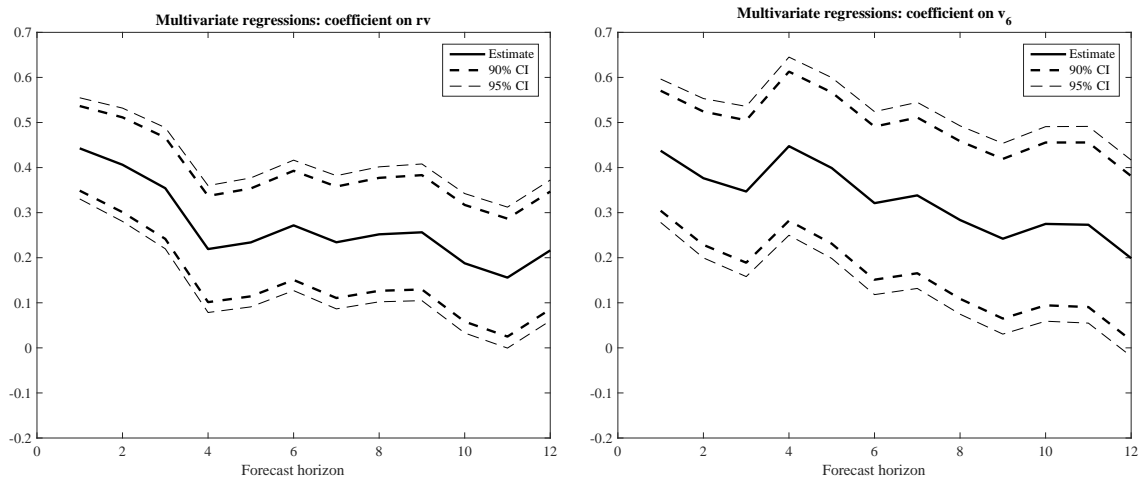
Wu, Jing Cynthia and Fan Dora Xia, “Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound,” 2014. Working paper.

Figure 1: Time series of realized volatility and expectations



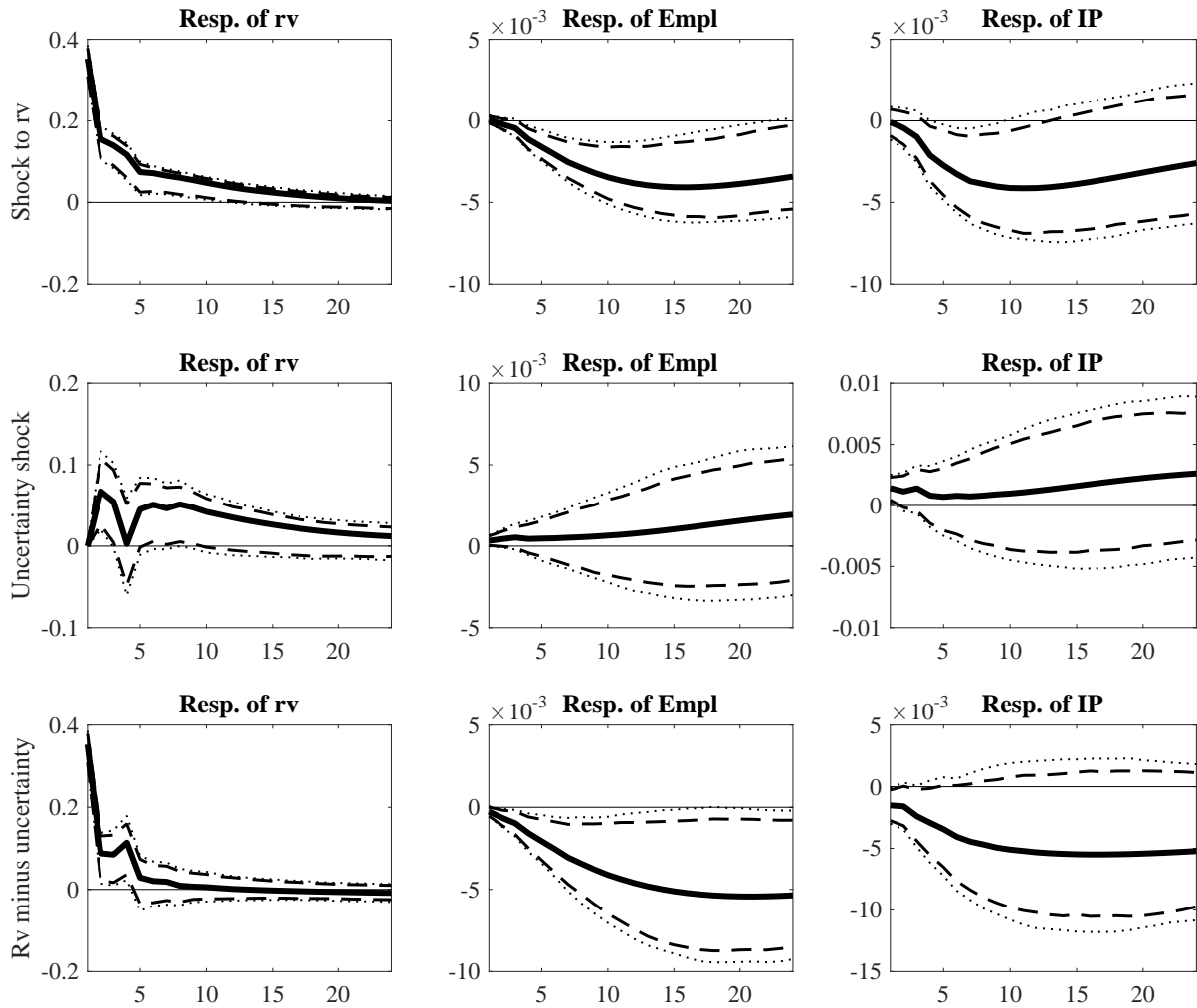
Note: Time series of realized volatility (RV), and 6-month market-implied uncertainty (V_6), in annualized units. Grey bars indicate NBER recessions.

Figure 2: Predictive regressions



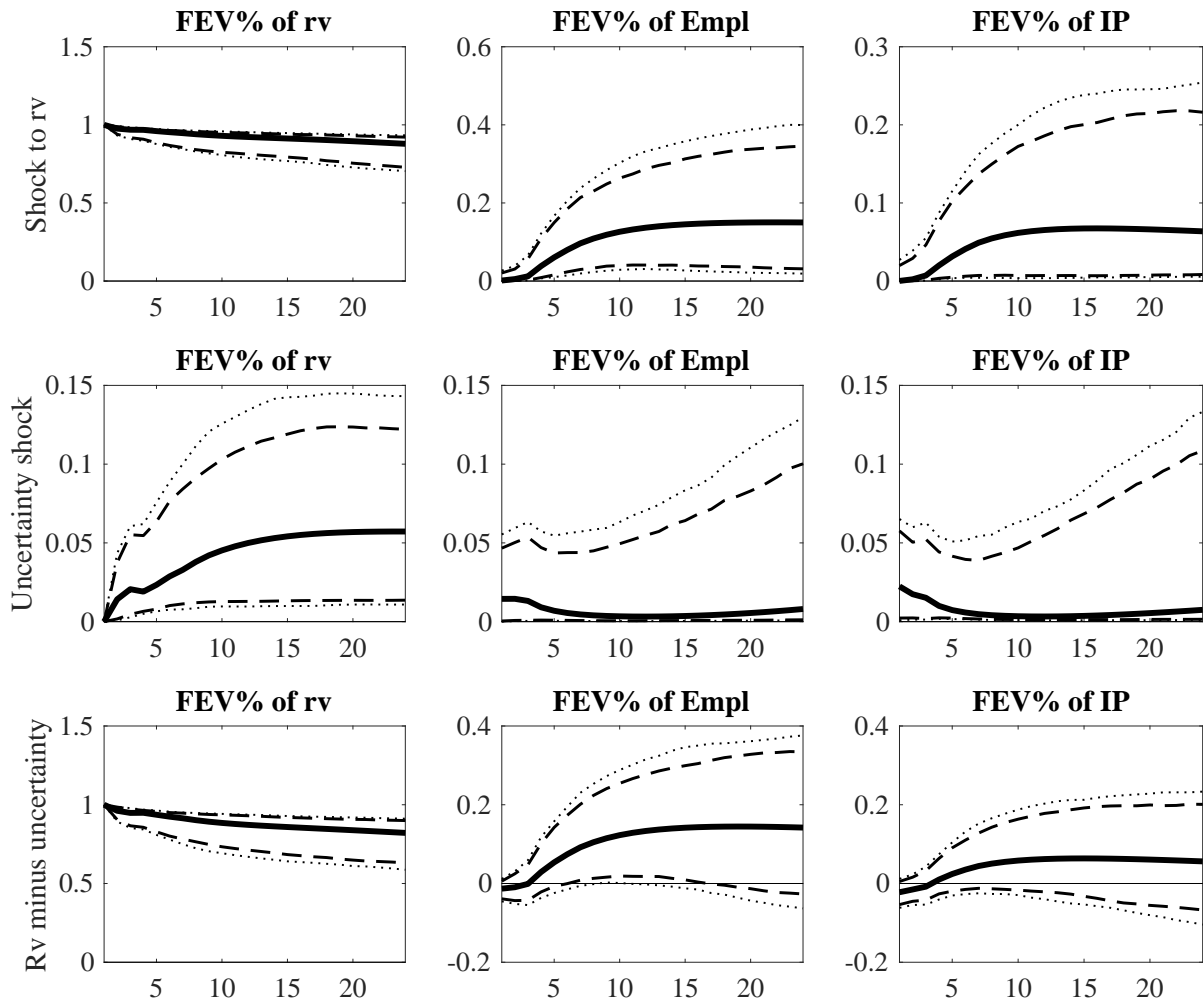
Note: Coefficients of multivariate regressions of realized volatility rv on lagged rv and lagged market-implied uncertainty v_6 , for different lags (on the x-axis). For each lag, the left panel shows the coefficient on rv , the right panel the coefficient on v_6 .

Figure 3: Impulse response functions from benchmark VAR



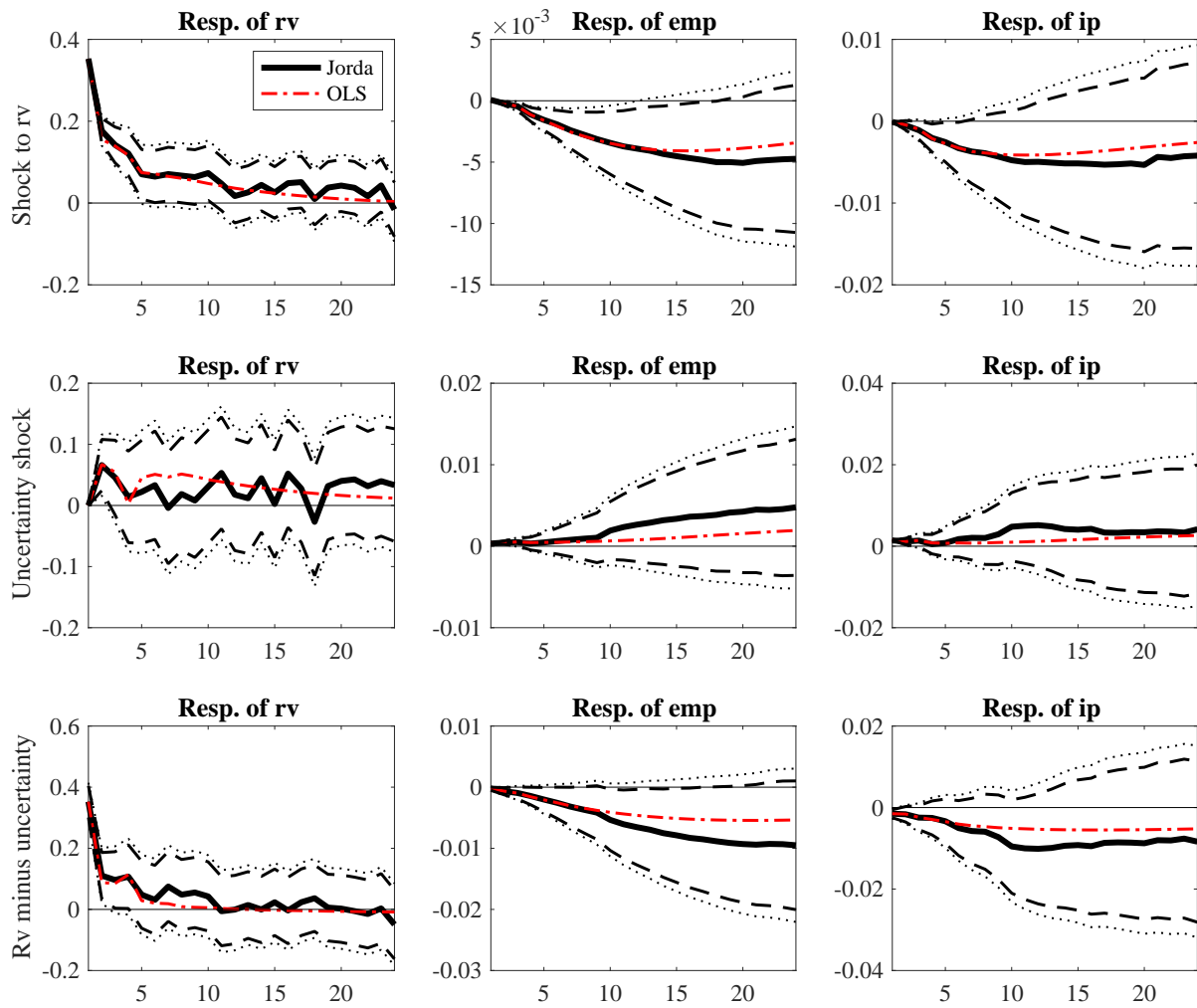
Note: Responses of rv , employment, and industrial production to shocks to rv and the identified uncertainty shock, in a VAR with rv , v_6 , federal funds rate, log employment, and log industrial production. The IRFs are scaled so that the two shocks have equal effects on rv over months 2–24 following the shock. The sample period is 1983–2014.

Figure 4: Forecast error variance decomposition



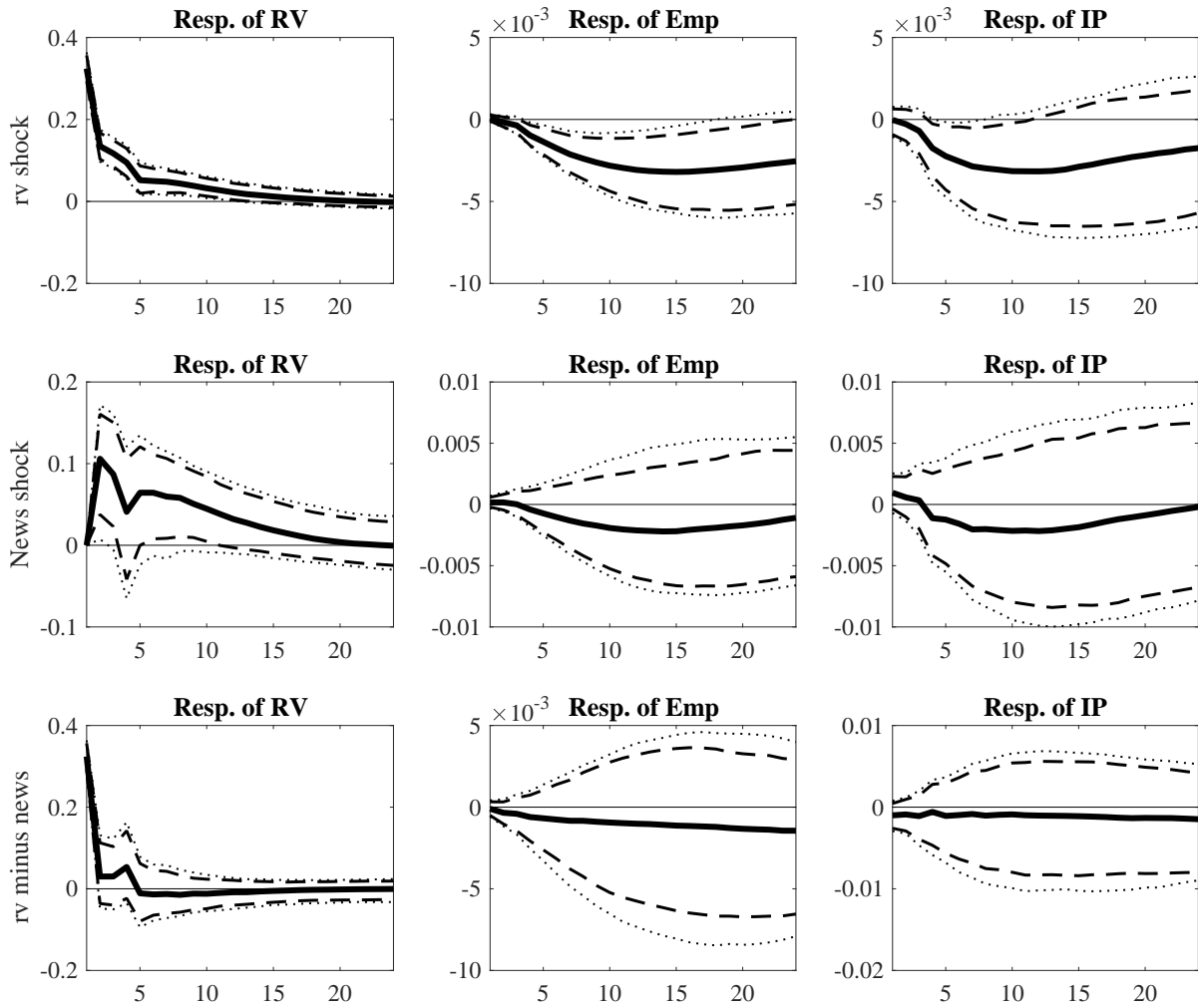
Note: Fraction of the forecast error variance (FEV) of rv , employment, and industrial production to shocks to rv and uncertainty in the VAR of figure 3. The figure also reports 90% and 95% confidence intervals.

Figure 5: Jorda local projections



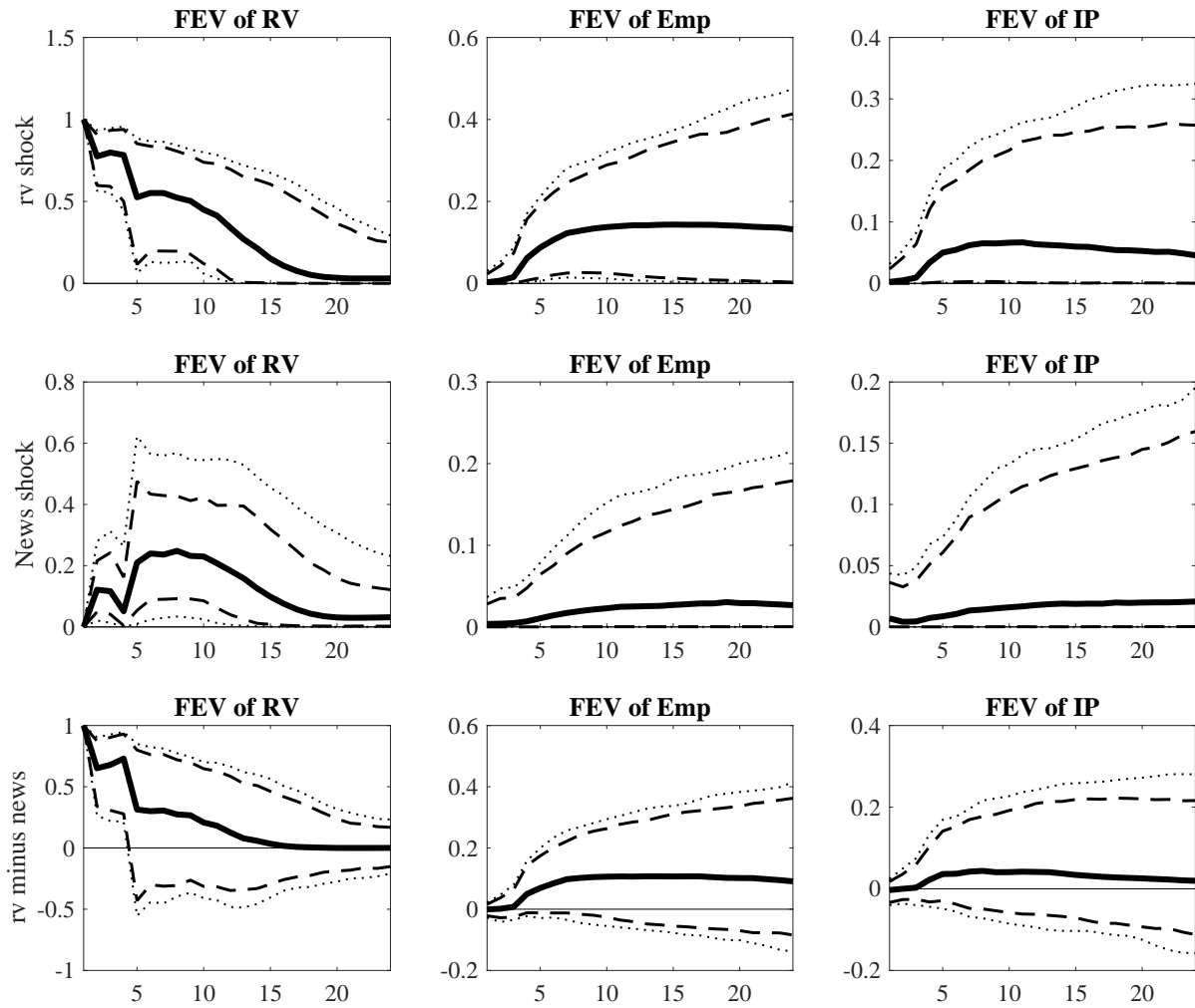
Note: Impulse response functions from local projections (Jorda (2005)), with 90% and 95% CI, of *rv*, employment, and industrial production to shocks to *rv* and uncertainty, in a VAR with *rv*, *v₆*, federal funds rate, log employment and log industrial production. The figure also reports the VAR-based impulse-response function for comparison.

Figure 6: Impulse response functions allowing *slope* to enter



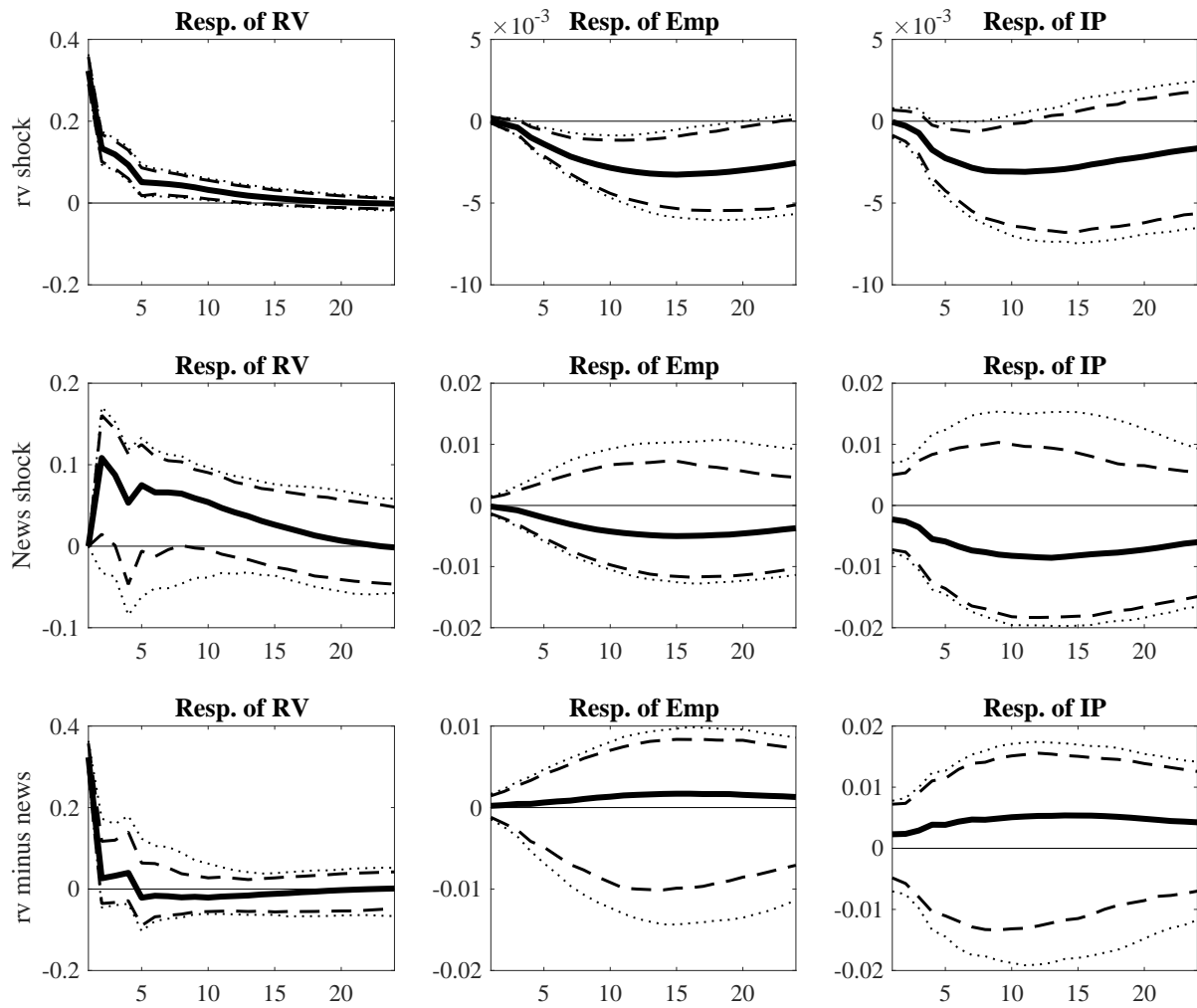
Note: See figure 3. In this case, *slope* is also included in the VAR and allowed to help forecast future *rv* in the construction of the news shock.

Figure 7: Forecast error variance decomposition allowing *slope* to enter



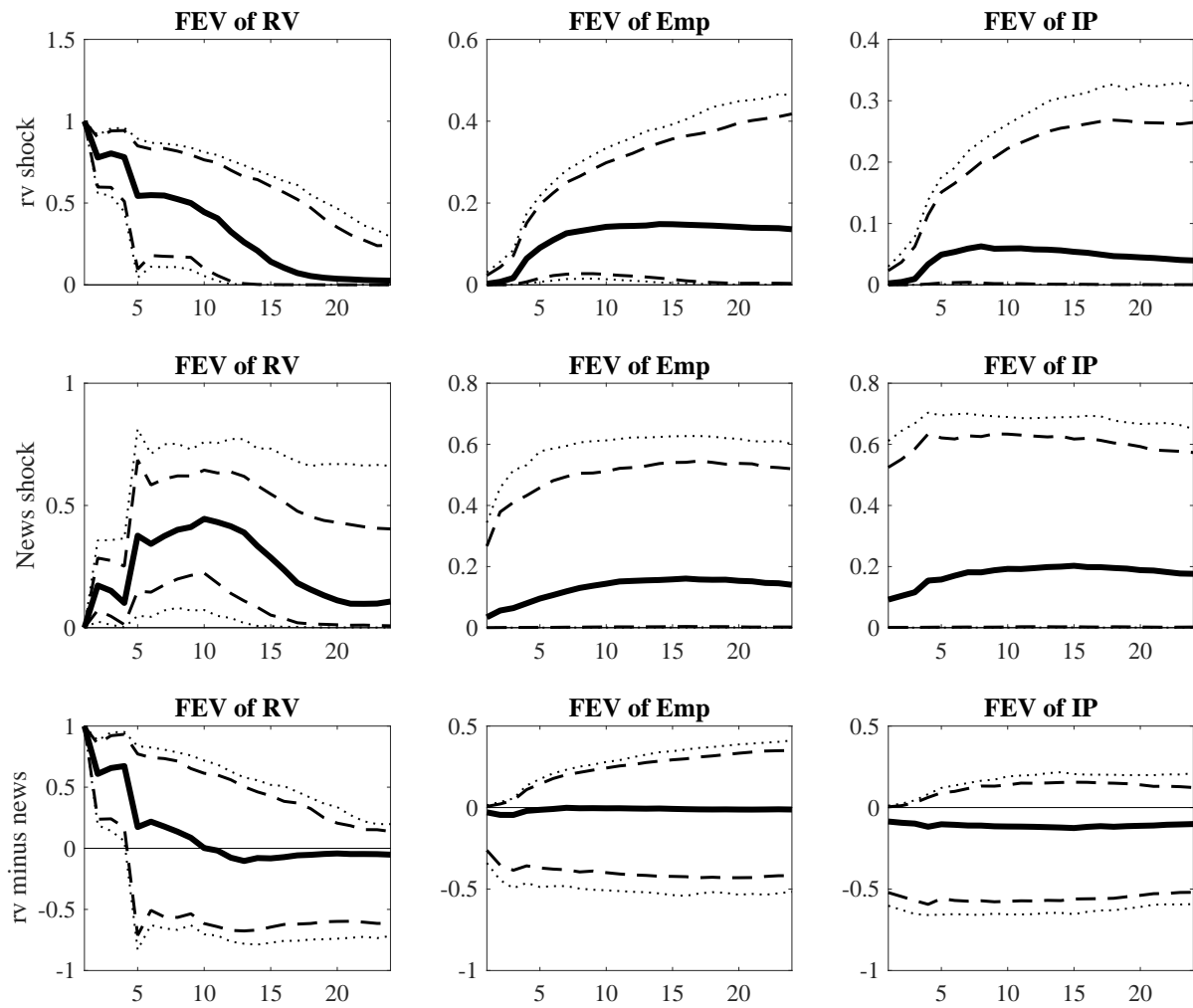
Note: See figure 4. *slope* is included in the VAR as part of the news shock.

Figure 8: Impulse response functions using unrestricted BBL identification



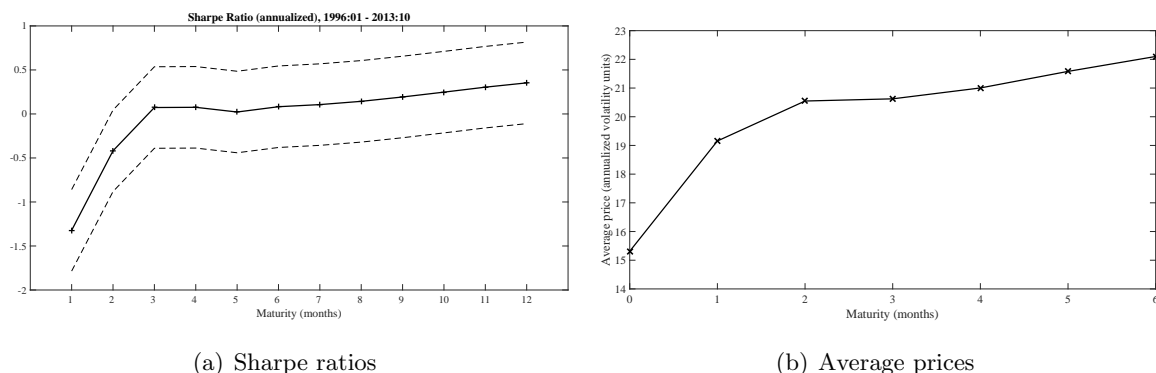
Note: See figure 3. All variables in the VAR may now help forecast *rv*.

Figure 9: Forecast error variance decomposition using unrestricted BBL identification



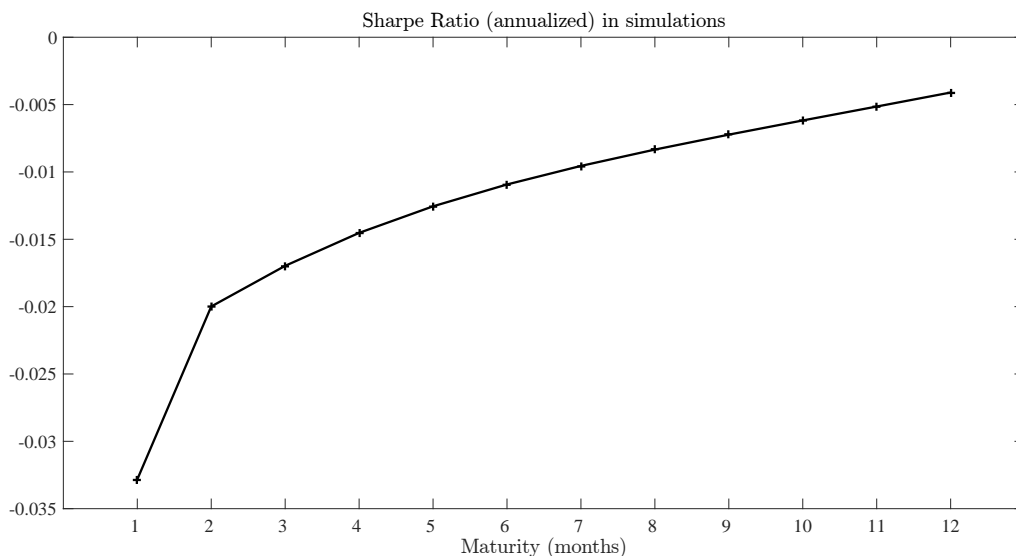
Note: See figure 4. All variables in the VAR may now help forecast *rv*.

Figure 10: Forward variance claims: returns and prices



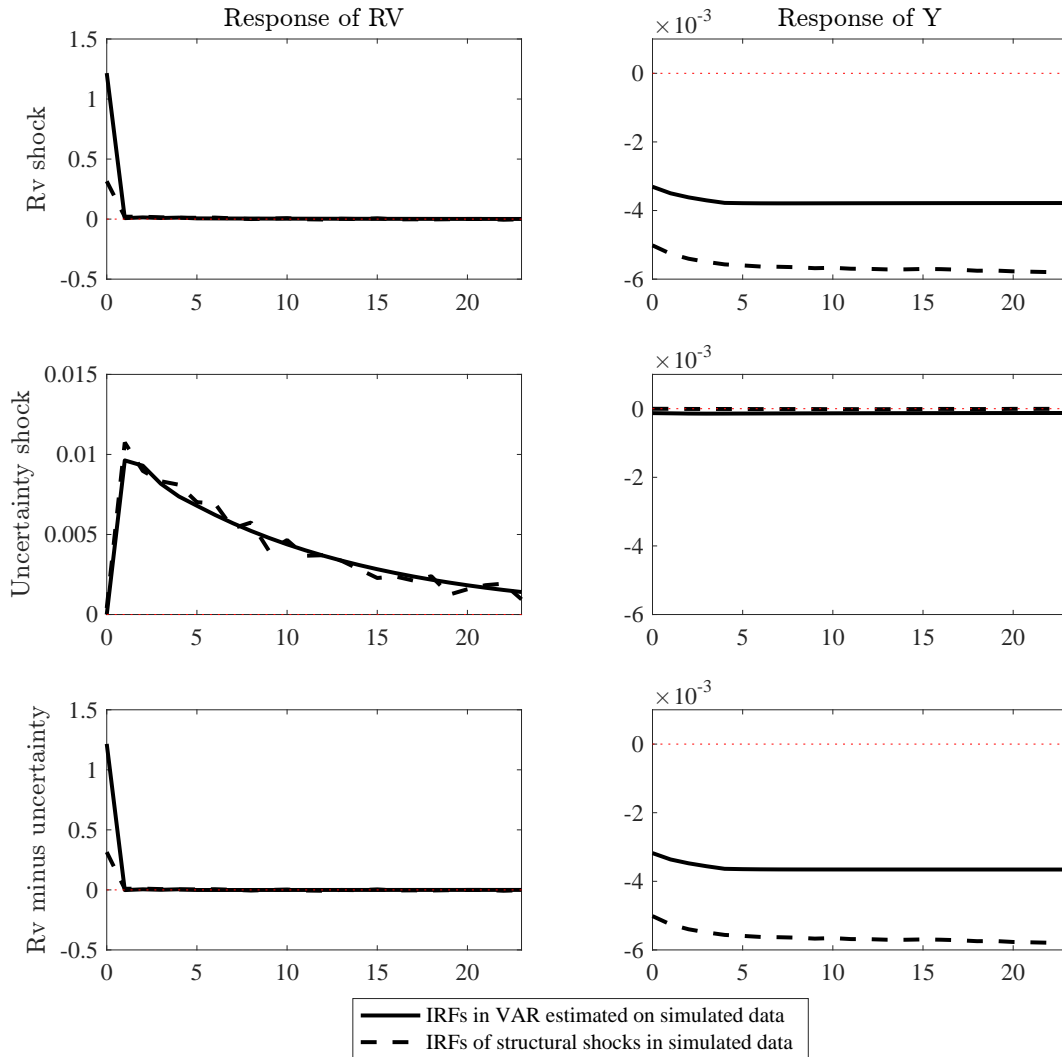
Note: Panel A shows the annualized Sharpe ratio for the forward variance claims, constructed using Variance Swaps. The returns are calculated assuming that the investment in an n-month variance claim is rolled over each month. Dotted lines represent 95% confidence intervals. All tests for the difference in Sharpe ratio between the 1-month variance swap and any other maturity confirm that they are statistically different with a p-value of 0.03 (for the second month) and < 0.01 (for all other maturities). The sample used is 1996-2013. For more information on the data sources, see Dew-Becker et al. (2015). Panel B shows the average prices of forward variance claims of different maturity, constructed from option prices, for the period 1983–2014. All prices are reported in annualized volatility terms. Maturity zero corresponds to average realized volatility.

Figure 11: Annual Sharpe ratios on forward claims (simulated structural model)



Note: The figure shows annual Sharpe ratio on forward variance claims of maturity 1 to 12 months, in the simulated model of section 7.3. The Sharpe ratios are constructed as in Figure 10.

Figure 12: IRFs from structural model



Note: The figure shows impulse response functions from data simulated from the model in Section 7.3. Solid lines correspond to IRFs estimated using our VAR methodology as in Figure 3. Dashed lines correspond to IRFs for the two structural shocks $J\nu_t$ and η_t .

Table 1: Descriptive statistics

Panel A: Descriptive statistics					
	Mean	Std.	Skewness		
RV	15.31	9.34	3.62		
V_6	20.78	7.29	1.43		

Panel B: Correlations					
	rv	v_6	Unempl.	Cap. Ut.	S&P 500 ret.
rv	1.00	0.77	0.17	-0.33	-0.20
Uncertainty (v_6)	0.77	1.00	0.06	-0.38	-0.14
Unemployment	0.17	0.06	1.00	-0.70	0.10
Capacity Utilization	-0.33	-0.38	-0.70	1.00	-0.06
S&P 500 return	-0.20	-0.14	0.10	-0.06	1.00

Note: The table reports various statistics on realized volatility, expectations and their relationship. Panel A reports the mean, standard deviation and skewness of realized volatility and market-implied uncertainty v_6 . Panel B reports the correlations between those variables and with macroeconomic and financial variables: unemployment, capacity utilization, and the S&P 500 return. Sample period is 1983-2014.

Table 2: Predictability of rv

Predictors	(1)	(2)	(3)	(4)	(5)
rv	0.34*** (0.08)	0.22** (0.09)	0.22** (0.09)	0.22** (0.09)	0.19** (0.09)
v_6	0.39*** (0.15)	0.53*** (0.14)	0.42** (0.20)	0.53*** (0.14)	0.54*** (0.15)
$slope_t$		0.39 (0.25)	0.31 (0.26)	0.41* (0.25)	0.25 (0.25)
rv_{t-1}			0.09 (0.07)		
FFR				0.01 (0.02)	
$Empl$				-0.02 (1.39)	
IP				0.01 (0.94)	
PC 1					-0.010 (0.008)
PC 2					-0.018 (0.012)
PC 3					0.009 (0.007)
Rm					-0.34 (0.38)
N	377	377	377	377	377
Adj. R^2	0.42	0.43	0.43	0.44	0.45

Note: The figure reports the results of linear predictive regressions of 6-month realized volatility on lagged rv , volatility market prices (level and slope of the variance swap term structure) and various macroeconomic variables, with Hansen-Hodrick standard errors using a 6-month lag window.

Table 3: Sums of coefficients on lags of rv and v_6

Panel A: Full Sample		VAR specification	rv	v_6	diff.	p-value
Effect on:	Empl.	Empl., rv , v_6	-0.14	0.10	-0.24	0.06
	IP	IP, rv , v_6	-0.23	0.18	-0.41	0.31
	Empl.	IP, Empl., rv , v_6	-0.14	0.11	-0.25	0.05
	IP	IP, Empl., rv , v_6	-0.19	0.10	-0.29	0.52

Panel B: 1988-2006		VAR specification	rv	v_6	diff.	p-value
Effect on:	Empl.	Empl., rv , v_6	-0.16	0.10	-0.27	0.17
	IP	IP, rv , v_6	-0.22	0.11	-0.32	0.58
	Empl.	IP, Empl., rv , v_6	-0.15	0.09	-0.24	0.21
	IP	IP, Empl., rv , v_6	-0.12	0.05	-0.06	0.91

Note: The table reports the sum of the coefficients on the lags of rv and uncertainty v_6 in the equations for IP and Employment in VAR specifications that include rv , v_6 , as well as IP or Employment (or both). Columns 1 and 2 report the sums of the coefficients on the lags of rv and v_6 , respectively; column 3 reports the difference in the sums, and column 4 reports p-values for tests of this difference. Panel A performs the analysis on the full sample (1983–2014), while Panel B restricts the sample to 1988–2006.

Table 4: Skewness

Panel A: real economic activity	Monthly	Quarterly	Start of sample (year)
Employment	-0.41	-0.41	1948
Capacity Utilization	-1.02	-1.30	1967
IP	0.17	-0.16	1948
IP, starting 1960	-0.93	-1.28	1960
Y		-0.11	1947
C		-0.28	1947
I		-0.03	1947

Panel B: skewness of S&P 500 monthly returns	
Implied (since 1990)	-1.81
Realized (since 1926)	0.36
Realized (since 1948)	-0.42
Realized (since 1990)	-0.61

Note: Panel A reports the skewness of changes of employment, capacity utilization, industrial production (beginning both in 1948 and in 1960), GDP, consumption and investments. The first column reports the skewness of monthly changes, the second column the skewness of quarterly changes. Panel B reports the realized skewness of S&P 500 monthly returns in different periods, as well as the implied skewness computed by the CBOE using option prices.

Table 5: Model Calibration

<i>Panel A: Moments</i>	Model			Data		
	Mean	Std.	Skewness	Mean	Std.	Skewness
Returns	0.08	2.54	-3.41	7.46	14.77	-0.48
Output	2.00	1.94	-2.14	1.33	1.92	-0.11
Investment	2.00	3.79	-2.05	2.35	7.43	-0.03
Consumption	2.00	1.56	-1.67	1.2	1.06	-0.28

<i>Panel B: Corr. of VAR and structural shocks</i>		Structural shocks		
		Jv_t	η_t	ε_t
	RV	0.81	0.00	0.06
VAR identified shocks	Uncertainty	-0.07	0.96	-0.17

Note: Panel A reports the mean, standard deviation, and skewness of financial and macroeconomic variables in the data and in the model. Panel B shows the correlation between the structural shocks in the model and the shocks identified in the VAR.

Table 6: Predictability of rv using different measures of uncertainty

Predictors	(1)	(2)	(3)	(4)
rv	0.34*** (0.08)	0.59*** (0.07)	0.54*** (0.09)	0.31*** (0.08)
Option-based measure (v_6)	0.39*** (0.15)			0.53*** (0.16)
Baker, Bloom and Davis (2016)		-0.001 (0.001)		-0.0005 (0.001)
Michigan Survey			-0.009 (0.011)	-0.022 0.014
N	377	354	437	354
Adj. R^2	0.42	0.39	0.37	0.45

Note: The figure reports the results of linear predictive regressions of 6-month realized volatility on lagged rv , lagged 6-month implied volatility from option prices (v_6 , as in our baseline analysis), and two additional uncertainty measures: the Baker, Bloom and Davis (2016) measure and the Michigan Consumer Survey measure of the fraction of respondents who say they plan on delaying car purchases due to economic uncertainty. The table uses Hansen-Hodrick standard errors with a 6-month lag window.

A.1 Construction of option-implied volatility, V_n

In this section we describe the details of the procedure we use to construct model implied uncertainty at different horizons, starting from our dataset of end-of-day prices for American options on S&P 500 futures from the CME.

A.1.1 Main steps of construction of V_n

A first step in constructing the model-free implied volatility is to obtain implied volatilities corresponding to the observed option prices. We do so using a binomial model.¹ For the most recent years, CME itself provides the implied volatility together with the option price. For this part of the sample, the IV we estimate with the binomial model and the CME’s IV have a correlation of 99%, which provides an external validation on our implementation of the binomial model.

Once we have estimated these implied volatilities, we could in theory simply invert them to yield implied prices of European options on forwards. These can then be used to compute V_n directly as described in equation (11).

In practice, however, an extra step is required before inverting for the European option prices and integrating to obtain the model-free implied volatility. The model-free implied volatility defined in equation (11) depends on the integral of option prices over *all strikes*, but option prices are only observed at discrete strikes. We are therefore forced to interpolate option prices between available strikes and also extrapolate beyond the bounds of observed strikes.² Following the literature, we fit a parametric model to the Black–Scholes implied volatilities of the options and use the model to then interpolate and extrapolate across all strikes (see, for example, Jiang and Tian (2007), Carr and Wu (2009), Taylor, Yadav, and Zhang (2010), and references therein). Only after this extra interpolation-extrapolation step, the fitted implied volatilities are then inverted to yield option prices and compute V_n according to equation (11). To interpolate and extrapolate the implied volatility curve, we use the SVI (stochastic volatility inspired) model of Gatheral and Jacquier (2014).

In the next sections, we describe in more detail the interpolation-extrapolation step of the procedure (SVI fitting) as well as our construction of V_n after fitting the SVI curve. Finally, we report a description of the data we use and some examples and diagnostics on the SVI fitting method.

A.1.2 SVI interpolation: theory

There are numerous methods for fitting implied volatilities across strikes. Homescu (2011) provides a thorough review. We obtained the most success using Gatheral’s SVI model (see Gatheral and Jacquier 2014). SVI is widely used in financial institutions because it is parsimonious but also known

¹See for example Broadie and Detemple (1996) and Bakshi, Kapadia, and Madan (2003), among others.

²See Jiang and Tian (2007) for a discussion of biases arising from the failure to interpolate and extrapolate.

to approximate well the behavior of implied volatility in fully specified option pricing models (e.g. Gatheral and Jacquier (2011)); SVI also satisfies the limiting results for implied volatilities at very high and low strikes in Lee (2004), and, importantly, ensures that no-arbitrage conditions are not violated.

The SVI model simply assumes a hyperbolic relationship between implied variance (the square of the Black–Scholes implied volatility) and the log moneyness of the option, k (log strike/forward price).

$$\sigma_{BS}^2(k) = a + b \left(\rho(k - m) + \sqrt{(k - m)^2 + \sigma^2} \right)$$

where $\sigma_{BS}^2(k)$ is the implied variance under the Black–Scholes model at log moneyness k . SVI has five parameters: a , b , ρ , m , and σ . The parameter ρ controls asymmetry in the variances across strikes. Because the behavior of options at high strikes has minimal impact on the calculation of model-free implied volatilities, and because we generally observe few strikes far above the spot, we set $\rho = 0$ (in simulations with calculating the VIX for the S&P 500 – for which we observe a wide range of options – we have found that including or excluding ρ has minimal impact on the result).

We fit the parameters of SVI by minimizing the sum of squared fitting errors for the observed implied volatilities. Because the fitted values are non-linear in the parameters, the optimization must be performed numerically. We follow the methodology in Zeliade (2009) to analytically concentrate a and b out of the optimization. We then only need to optimize numerically over σ and m (as mentioned above, we set $\rho = 0$). We optimize with a grid search over $\sigma \times m = [0.001, 10] \times [-1, 1]$ followed by the simplex algorithm.

For many date/firm/maturity triplets, we do not have a sufficient number of contract observations to fit the implied volatility curve (i.e. sometimes fewer than four). We therefore include strike/implied volatility data from the two neighboring maturities and dates in the estimation. The parameters of SVI are obtained by minimizing squared fitting errors. We reweight the observations from the neighboring dates and maturities so that they carry the same amount of weight as the observations from the date and maturity of interest. Adding data in this way encourages smoothness in the estimates over time and across maturities but it does not induce a systematic upward or downward bias. We drop all date/firm/maturity triplets for which we have fewer than four total options with $k < 0$ or fewer than two options at the actual date/firm/maturity (i.e. ignoring the data from the neighboring dates and maturities).

When we estimate the parameters of the SVI model, we impose conditions that guarantee the absence of arbitrage. In particular, we assume that $b \leq \frac{4}{(1+|\rho|)T}$, which when we assume $\rho = 0$, simplifies to $b \leq \frac{4}{T}$. We also assume that $\sigma > 0.0001$ in order to ensure that the estimation is well defined. Those conditions do not necessarily guarantee, though, that the integral determining the model-free implied volatility is convergent (the absence of arbitrage implies that a risk-neutral probability density exists – it does not guarantee that it has a finite variance). We therefore eliminate observations where the integral determining the model-free implied volatility fails to converge numerically. Specifically, we eliminate observations where the argument of the integral

does not approach zero as the log strike rises above two standard deviations from the spot or falls more than five standard deviations below the strike (measured based on the at-the-money implied volatility).

A.1.3 Construction of V_n from the SVI fitted curve

After fitting the SVI curve for each date and maturity, we compute the integral in equation (11) numerically, over a range of strikes from -5 to +2 standard deviations away from the spot price.³ We then have V_n for every firm/date/maturity observation. The model-free implied volatilities are then interpolated (but not extrapolated) to construct V_n at maturities from 1–6 months for each firm/date pair.

A.1.4 Data description and diagnostics of SVI fitting

Our dataset consists of 2.3 million end-of-day prices for all American options on S&P 500 futures from the CME.

When more than one option (e.g. a call and a put) is available at any strike, we compute IV at that strike as the average of the observed IVs. We keep only IVs greater than zero, at maturities higher than 9 days and lower than 2 years, for a total of 1.9 million IVs. The number of available options has increased over time, as demonstrated by Figure A.2 (top panel), which plots the number of options available for V_n estimation in each year.

The maturity structure of observed options has also expanded over time, with options being introduced at higher maturities and for more intermediate maturities. Figure A.1 (top panel) reports the cross-sectional distribution of available maturities in each year to estimate the term structure of the model-free implied volatility. The average maturity of available options over our sample was 4 months, and was relatively stable. The maximum maturity observed ranged from 9 to 24 months and varied substantially over time.

Crucial to compute the model-free implied volatility is the availability of IVs at low strikes, since options with low strikes receive a high weight in the construction of V_n . The bottom panel of Figure A.1 reports the minimum observed strike year by year, in standard deviations below the spot price. In particular, for each day we computed the minimum available strike price, and the figure plots the average of these minimum strike price across all days in each year; this ensures that the number reported does not simply reflect outlier strikes that only appear for small parts of each year.

Figure A.1 shows that in the early part of our sample, we can typically observe options with strikes around 2 standard deviations below the spot price; this number increases to around 2.5 towards the end of the sample.

³In general this range of strikes is sufficient to calculate V_n . However, the model-free implied volatility technically involves an integral over the entire positive real line. Our calculation is thus literally a calculation of Andersen and Bondarenko's (2007) corridor implied volatility. We use this fact also when calculating realized volatility.

These figures show that while the number of options was significantly smaller at the beginning of the sample (1983), the maturities observed and the strikes observed did not change dramatically over time.

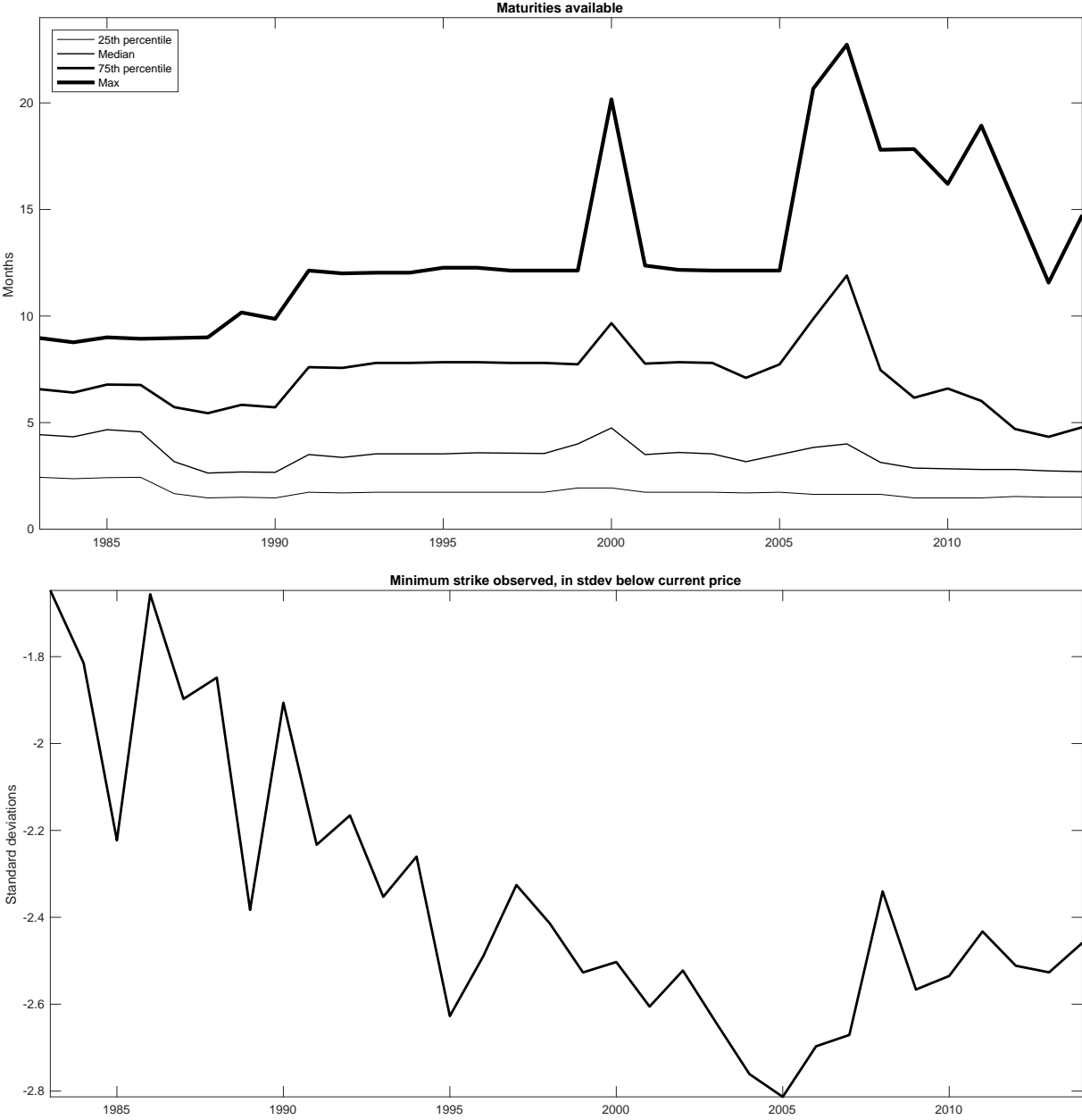
Figure A.3 shows an example of the SVI fitting procedure for a specific day in the early part of our sample (November 7th 1985). Each panel in the figure corresponds to a different maturity. On that day, we observe options at three different maturities, of approximately 1, 4, and 8 months. In each panel, the x's represent observed IVs at different values of log moneyness k . The line is the fitted SVI curve, that shows both the interpolation and the extrapolation obtained from the model.

Figure A.4 repeats the exercise in the later part of our sample (Nov. 1st 2006), where many more maturities and strikes are available.

Both figures show that the SVI model fits the observed variances extremely well. The bottom panel of Figure A.2 shows the average relative pricing error for the SVI model in absolute value. The graph shows that the typical pricing error for most of the sample is around 0.02, meaning that the SVI deviates from the observed IV by around 2% on average. Only in the very first years (up to 1985) pricing errors are larger, but still only around 10% of the observed IV.

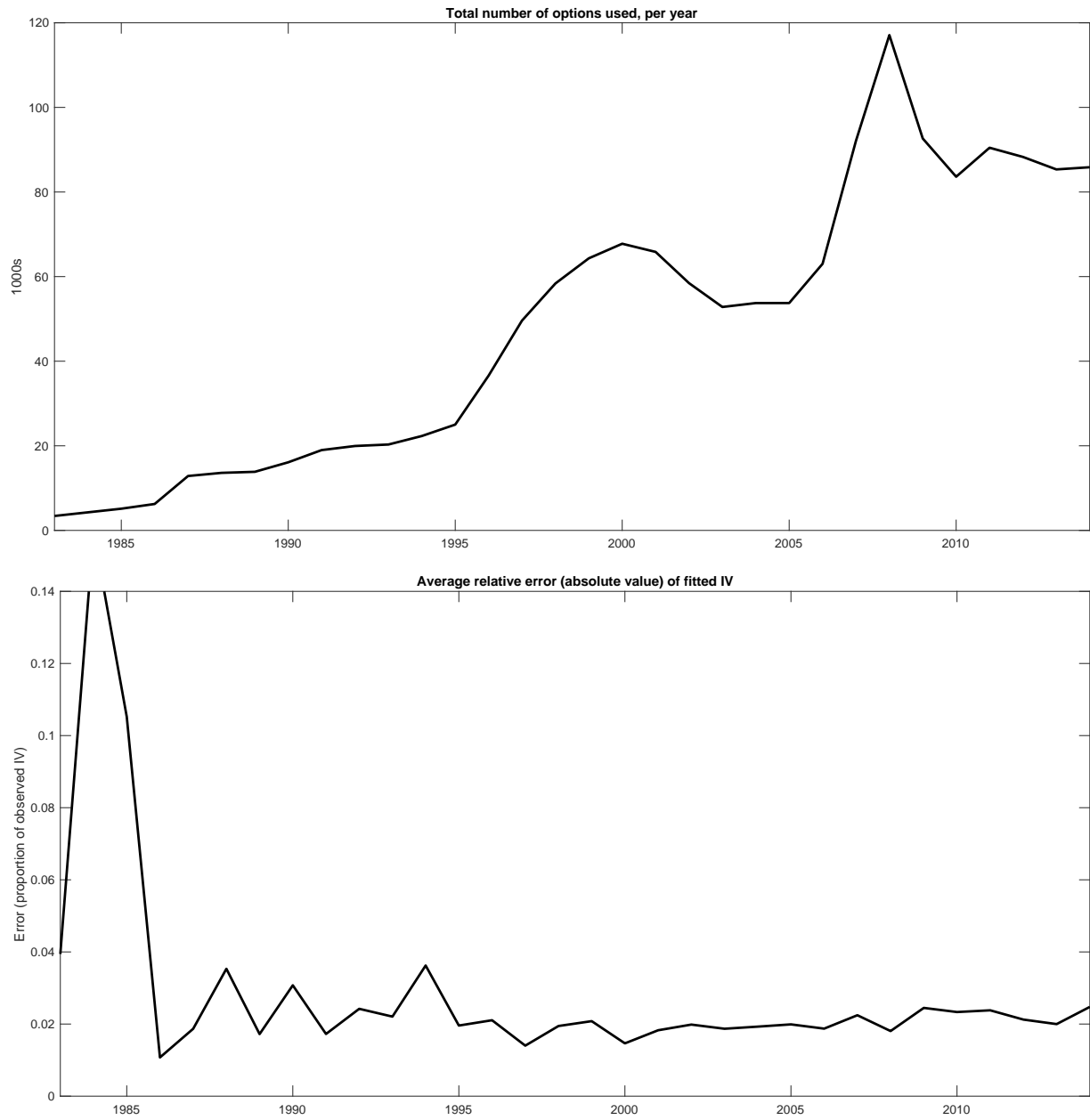
Overall, the evidence in this section shows that our observed option sample since 1983 has been relatively stable along the main dimensions that matter for our analysis – maturity structure, strikes observed, and goodness of fit of the SVI model.

Figure A.1: Maturities and strikes in the CME dataset



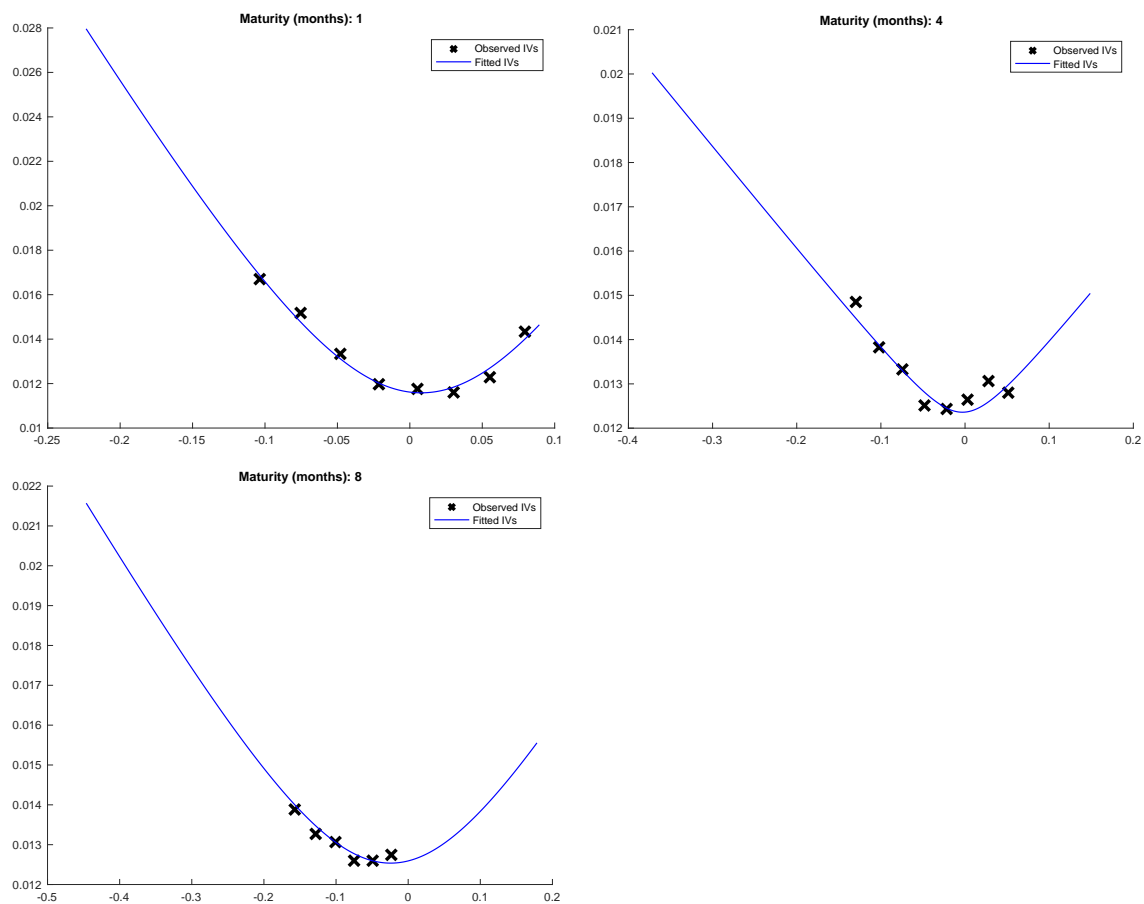
Note: Top panel reports the distribution of maturities of options used to compute the VIX in each year, in months. Bottom panel reports the average minimum strike in each year, in standard deviations below the forward price. The number is obtained by computing the minimum observed strike in each date and at each maturity (in standard deviations below the forward price), and then averaging it within each year to minimize the effect of outliers.

Figure A.2: Number of options to construct the VIX and pricing errors



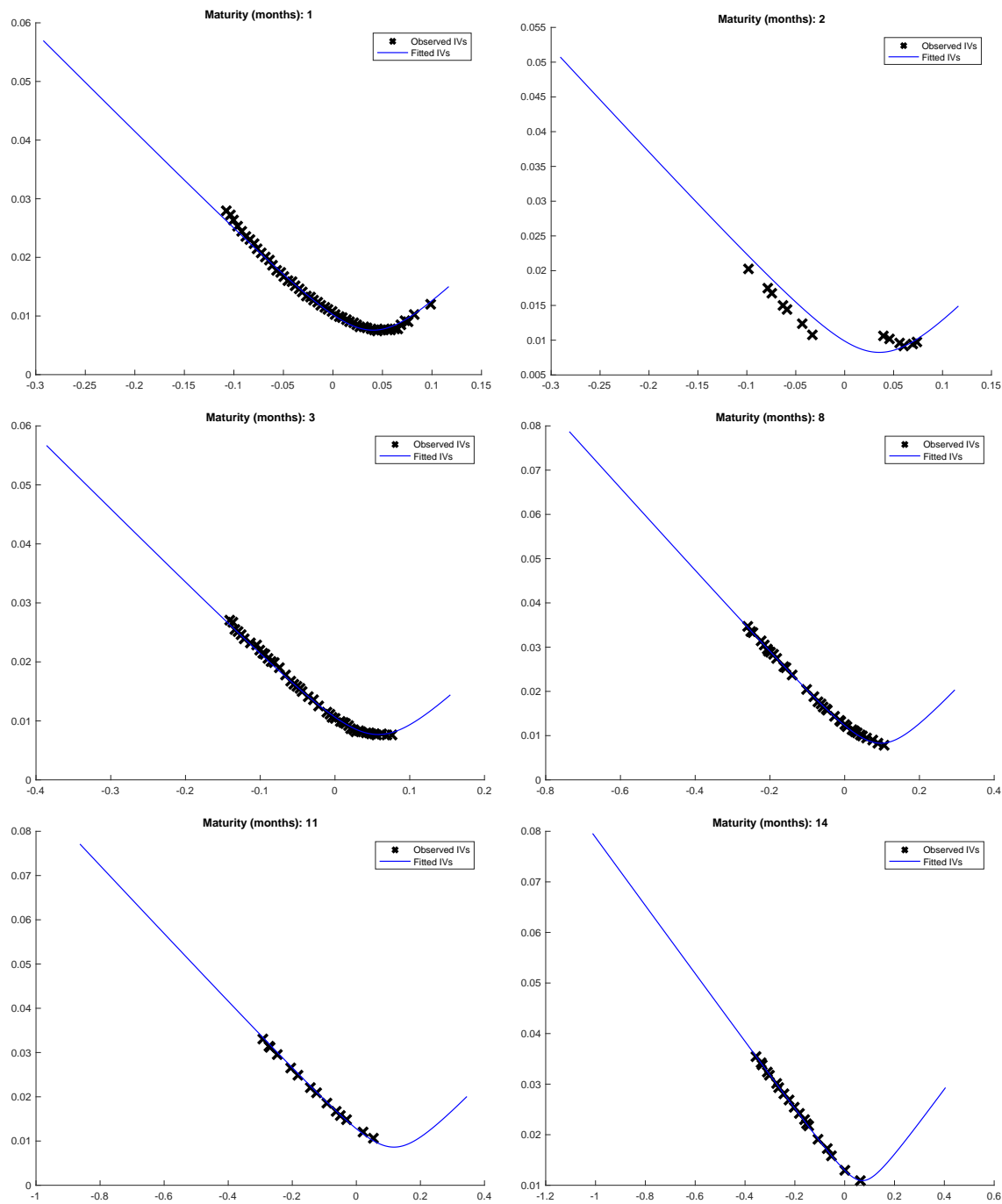
Note: Top panel reports the number of options used to compute the VIX in each year, in thousands. Bottom panel reports the average absolute value of the pricing error of the SVI fitted line relative to the observed implied variances, in proportional terms (i.e. 0.02 means absolute value of the pricing error is 2% of the observed implied variance).

Figure A.3: SVI fit: 11/7/1985



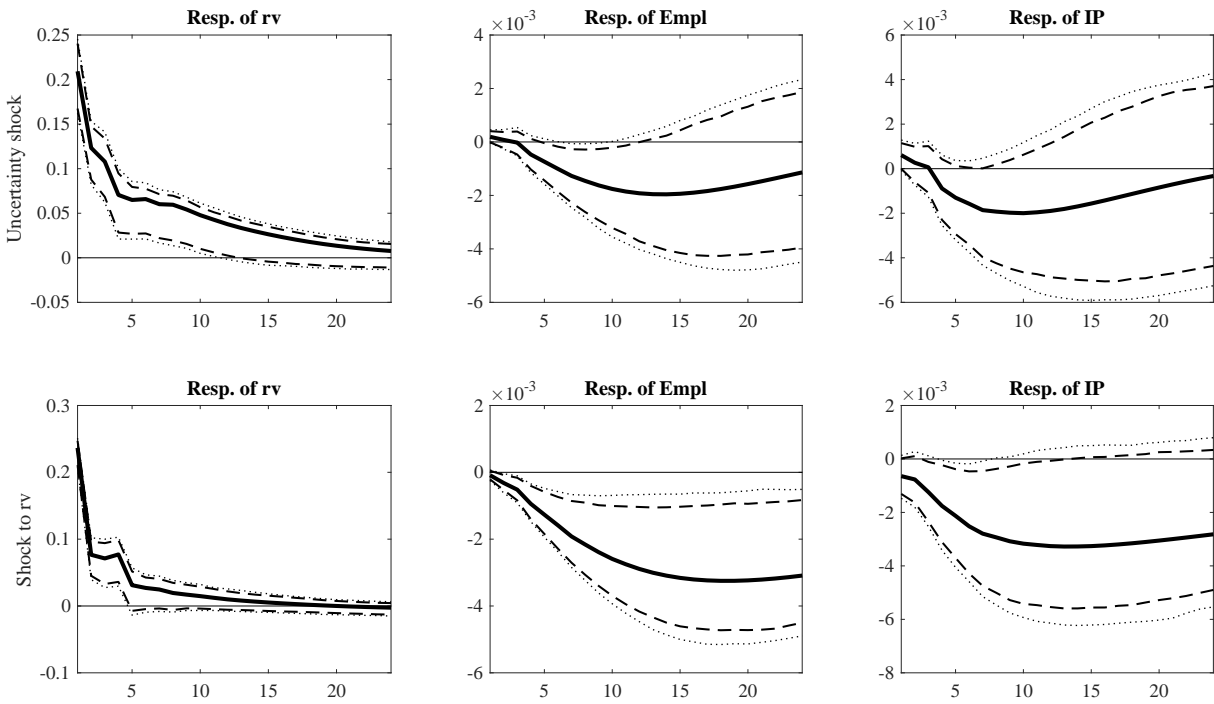
Note: Fitted implied variance curve on 11/7/1987, for the three available maturities. X axis is the difference in log strike and log forward price. x's correspond to the observed implied variances, and the line is the fitted SVI curve.

Figure A.4: SVI fit: 11/1/2006



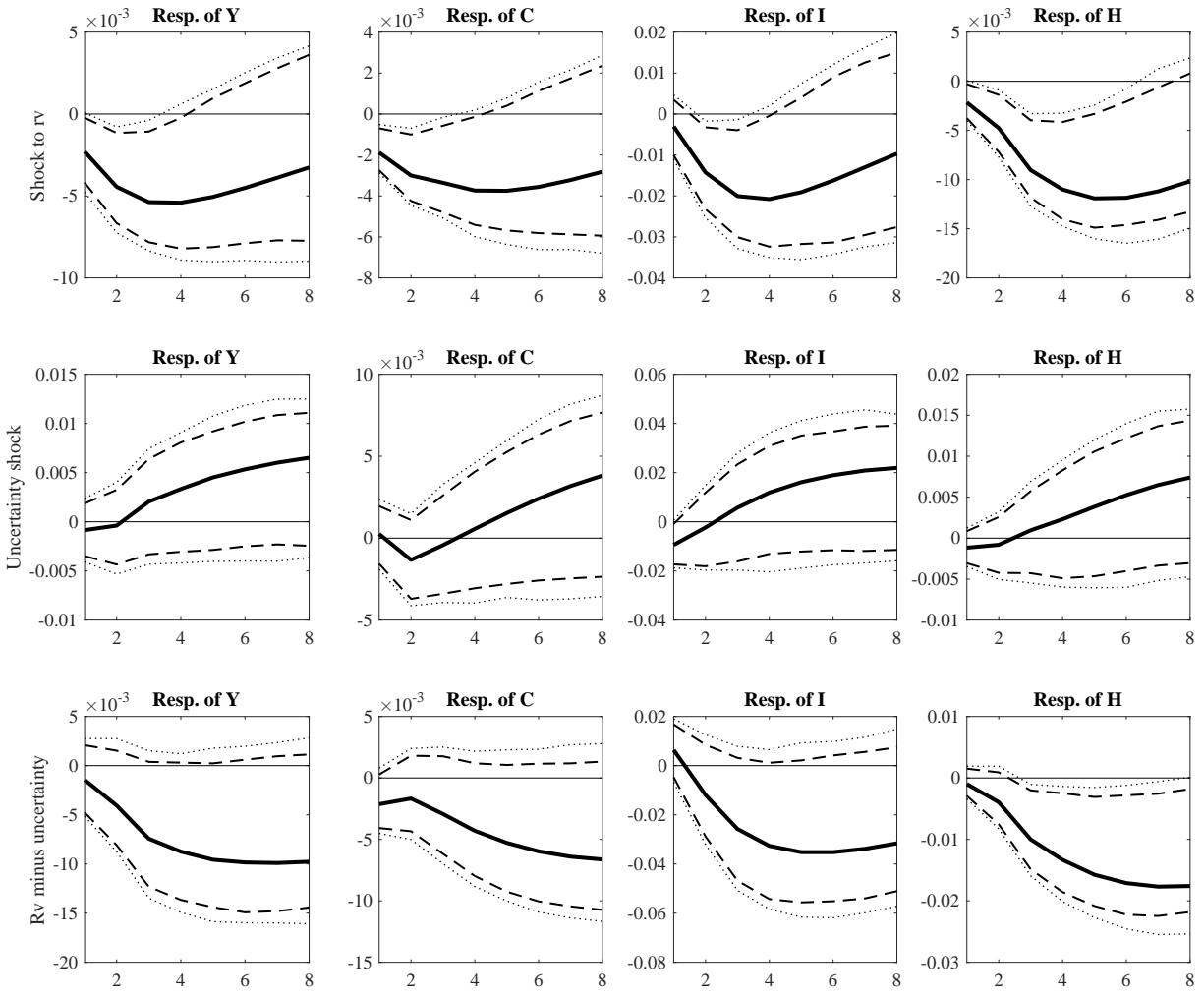
Note: Fitted implied variance curve on 11/1/2006, for the three available maturities. X axis is the difference in log strike and log forward price. x's correspond to the observed implied variances, and the line is the fitted SVI curve. On 11/1/2006 also a maturity of 5 months was available (not plotted for reasons of space).

Figure A.5: Impulse response functions from VAR ordering uncertainty first and rv second



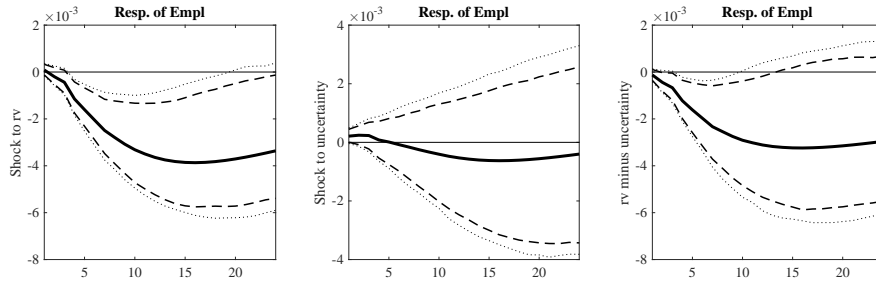
Note: See figure 3. Unlike in the baseline identification, the identified uncertainty shock is not orthogonalized with respect to rv . The rv shock in this case is the remaining part of reduced-form innovation to rv that is not spanned by the uncertainty shock.

Figure A.6: Quarterly VAR specification

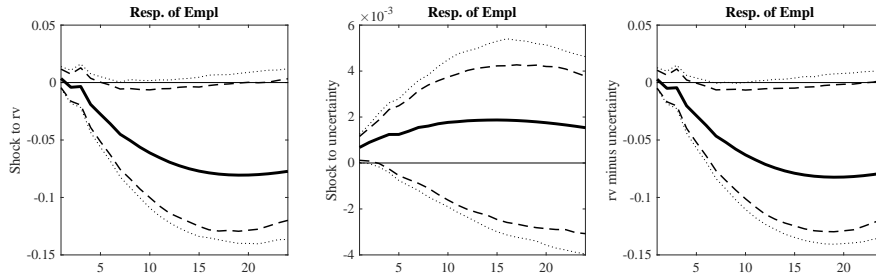


Note: See figure 3. Here we use the quarterly data from Basu and Bundick (2015) as the macro time series.

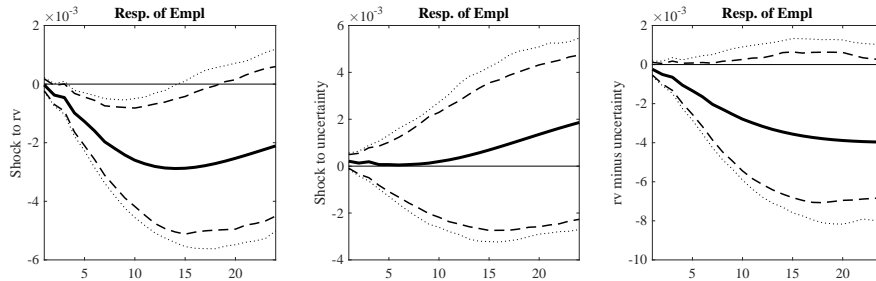
Figure A.7: Robustness (I): response of Employment to rv and volatility news shocks across specifications



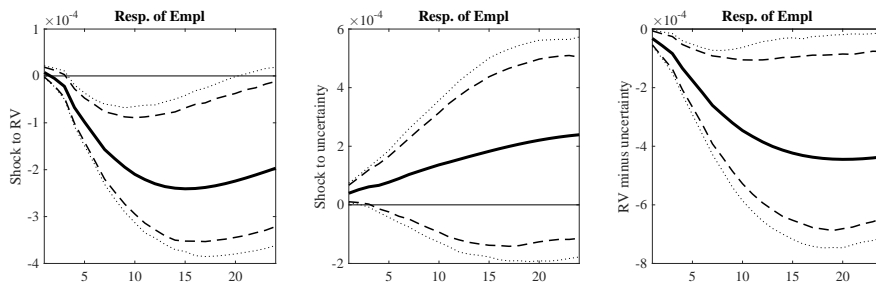
(a) Detrending the macroeconomic time series via HP filter



(b) Subperiod 1988-2006 (excluding 1987 crash and financial crisis)



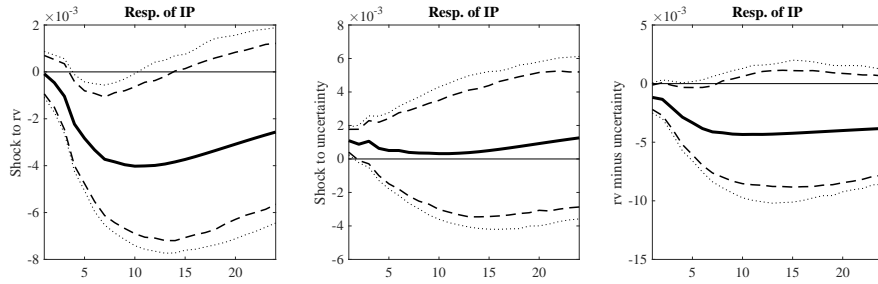
(c) Adding the S&P 500 level as first shock



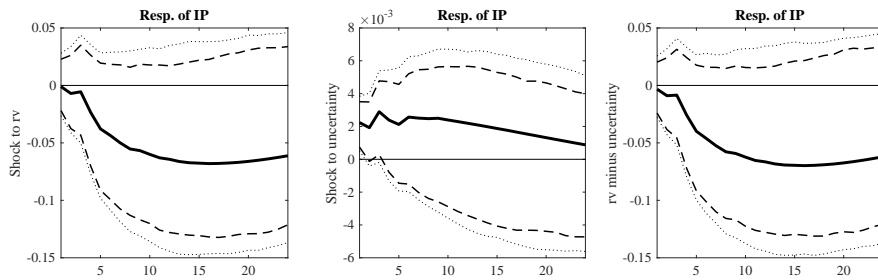
(d) Using RV and V_6 in levels, not logs

Note: Response of employment to RV shocks (left panels) and uncertainty (middle panels) with the difference in the right panel and different model specifications in each row. Row (a) detrends the macroeconomic time series via HP filter. Row (b) estimates the VAR in the subsample 1988-2006, which excludes both RV peaks (1987 crash and financial crisis). Row (c) orthogonalizes both the rv and the uncertainty shocks with respect to the reduced-form innovation in the S&P 500, as in Bloom (2009). Row (d) uses RV and V_6 in levels, not logs.

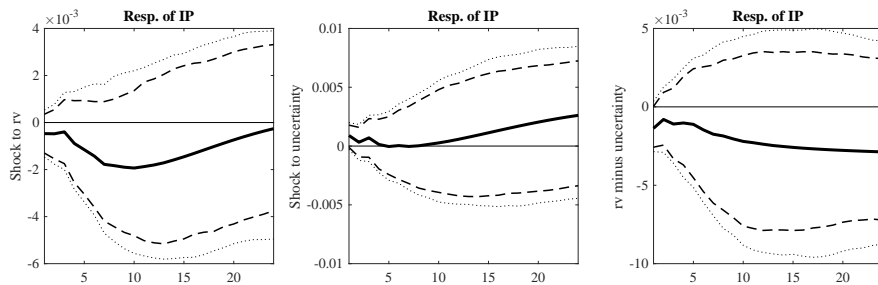
Figure A.8: Robustness (II): response of IP to rv and volatility news shocks across specifications



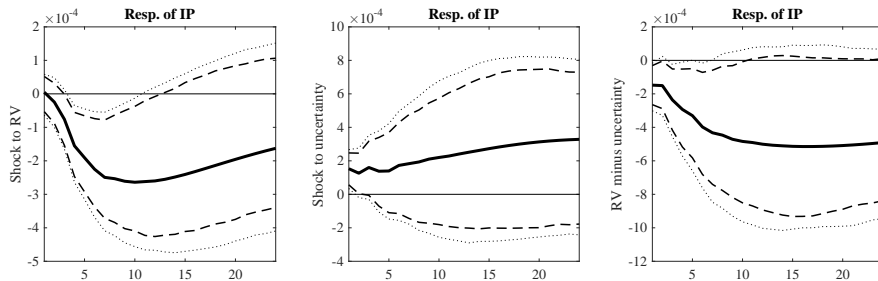
(a) Detrending the macroeconomic time series via HP filter



(b) Subperiod 1988-2006 (excluding 1987 crash and financial crisis)



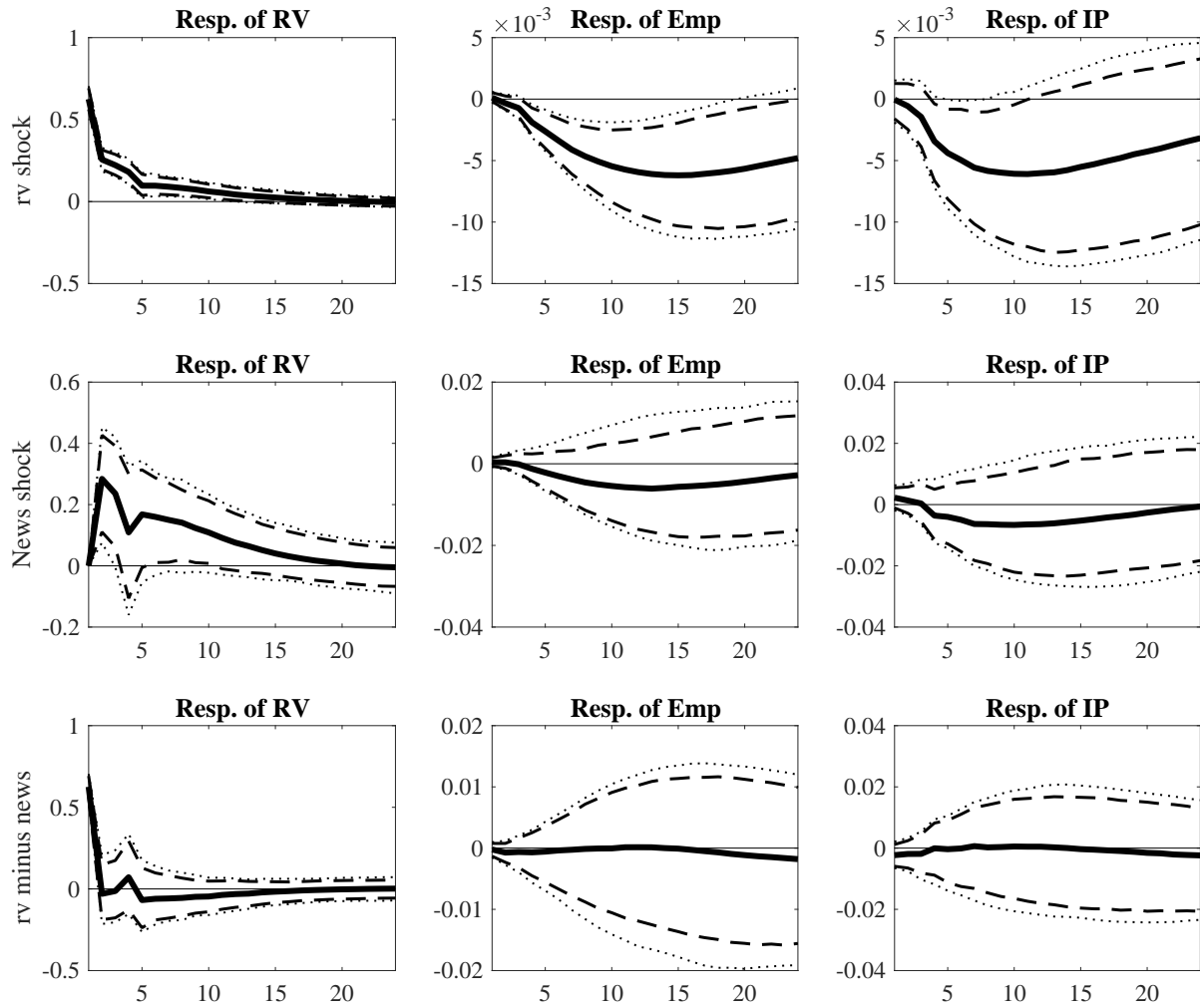
(c) Adding the S&P 500 level as first shock



(d) Using RV and V_6 in levels, not logs

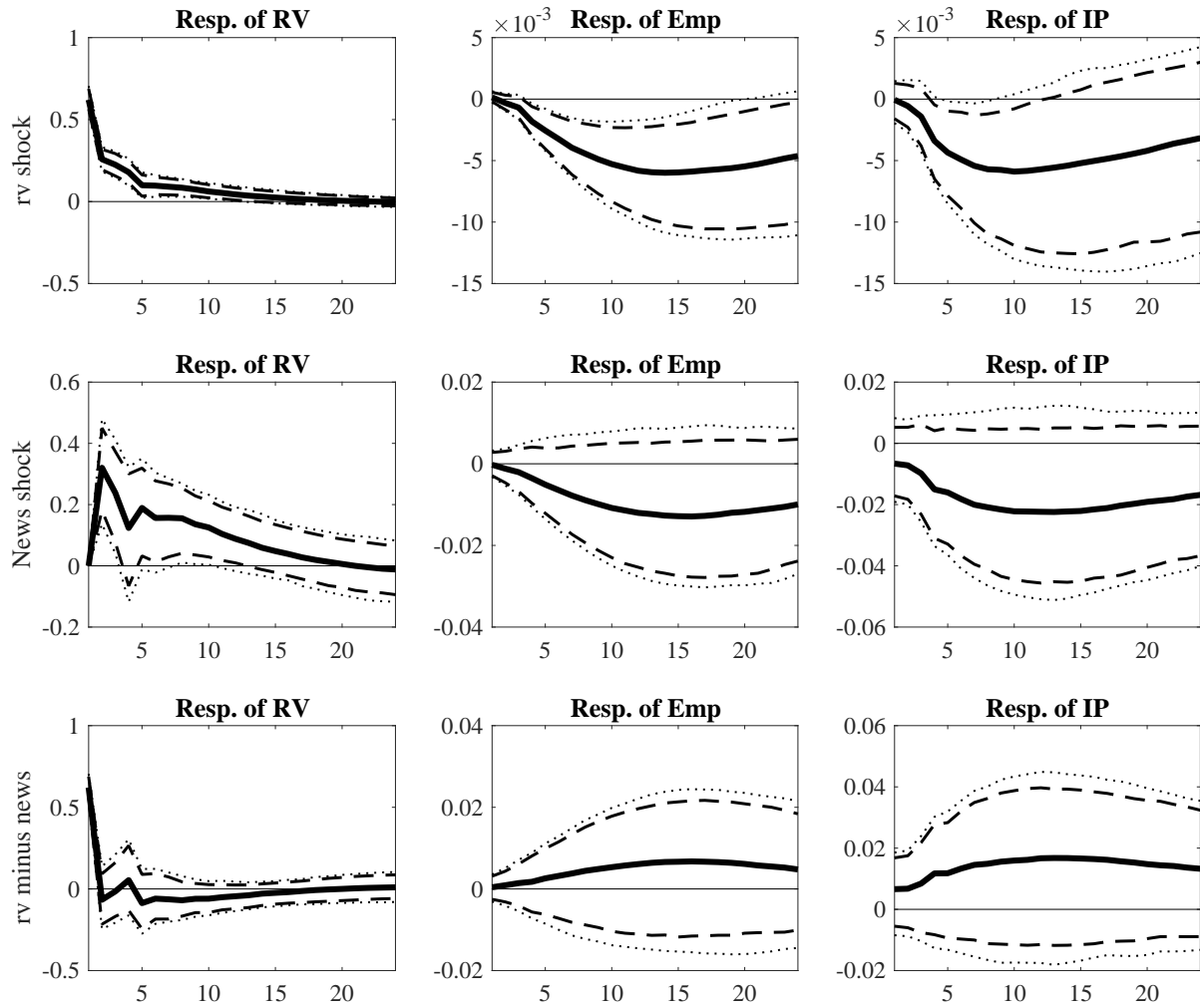
Note: See figure A.7. In this case the responses of IP are reported instead of employment.

Figure A.9: Impulse response functions from VAR including *slope* with identification horizon of 6 months



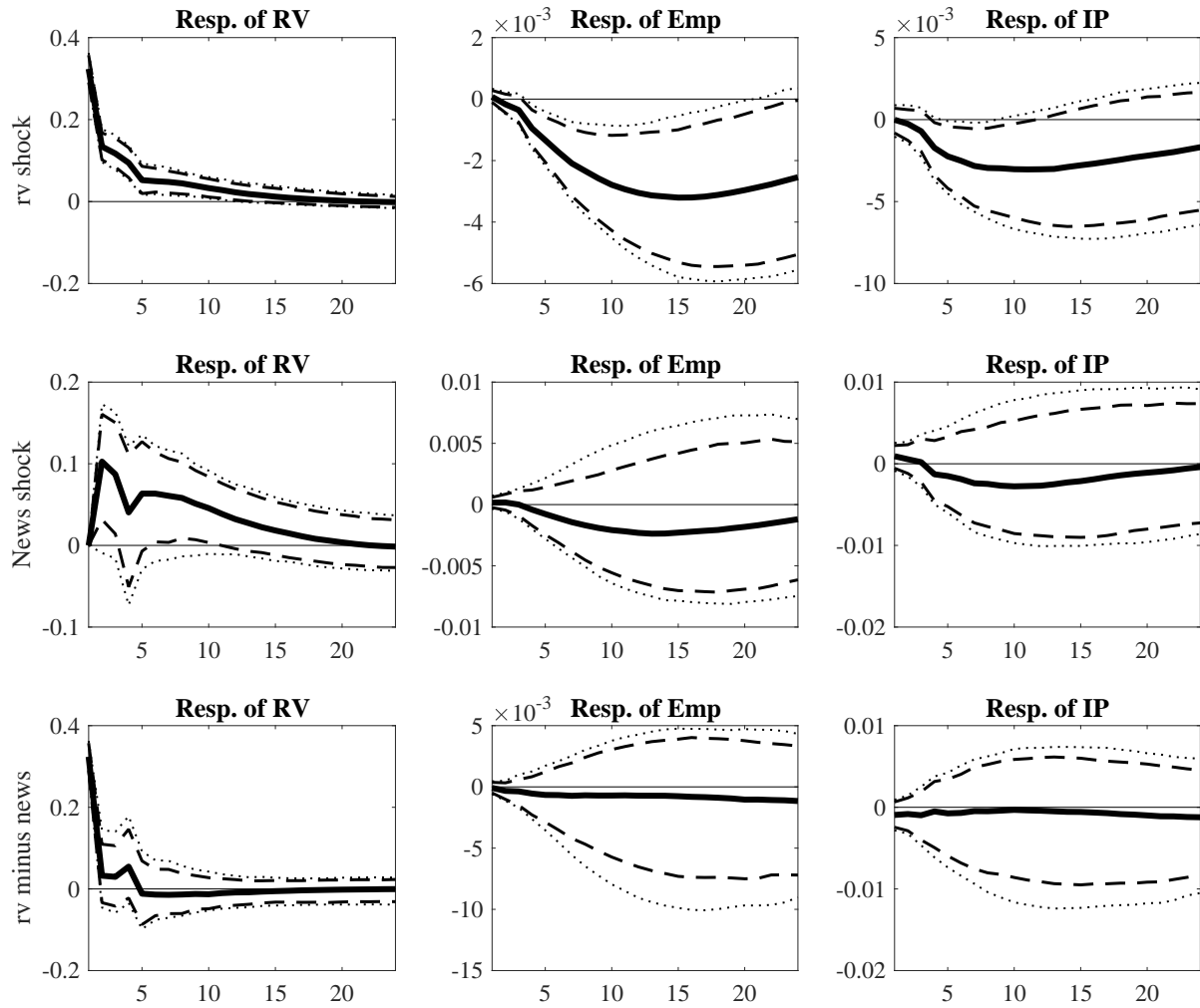
Note: See figure 6. The horizon used for identification of the news shock is now 6 months.

Figure A.10: Impulse response functions with unrestricted identification and identification horizon of 6 months



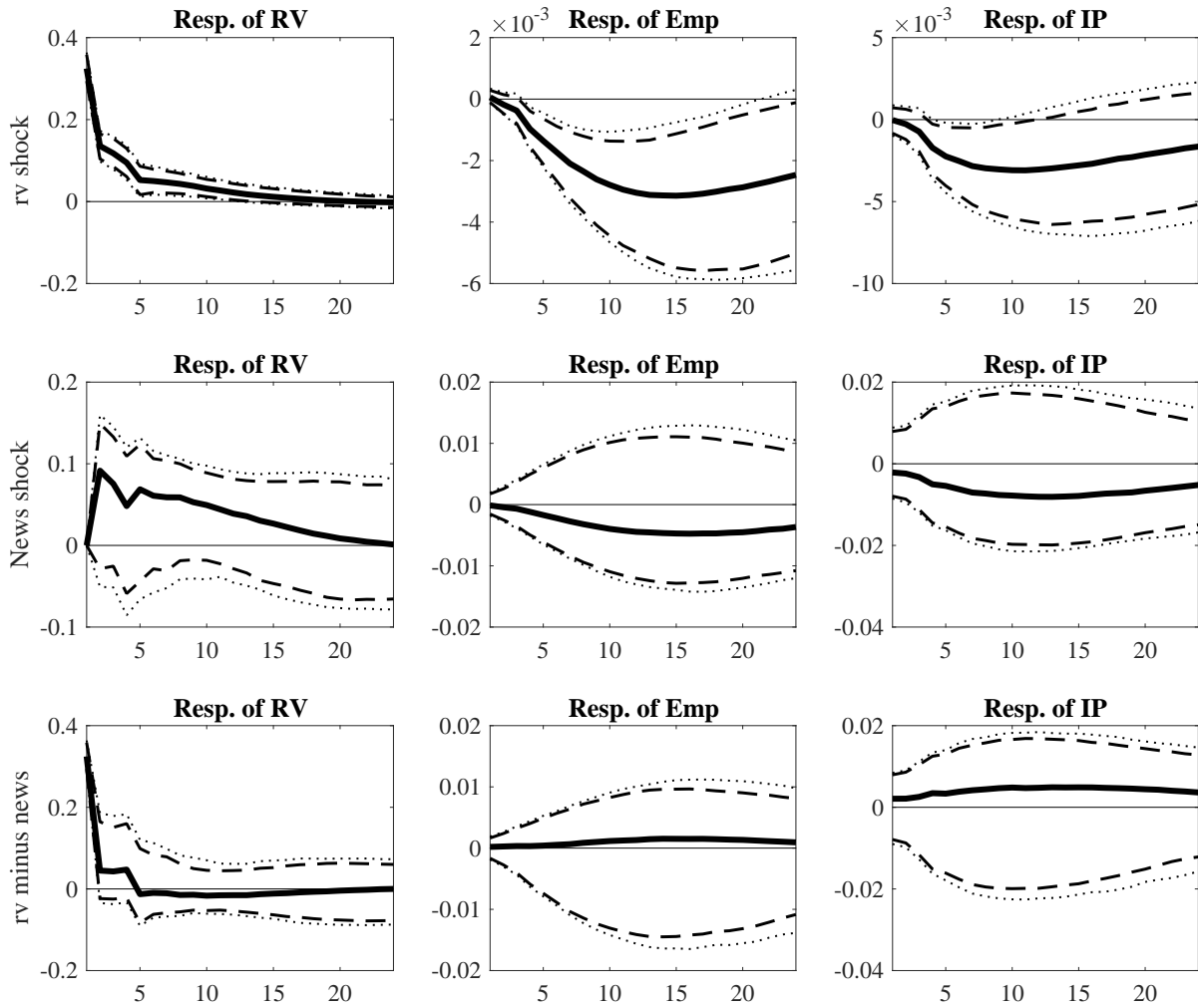
Note: See figure 8. The horizon used for identification of the news shock is now 6 months.

Figure A.11: Impulse response functions from VAR including *slope* with identification horizon of 48 months



Note: See figure 6. The horizon used for identification of the news shock is now 48 months.

Figure A.12: Impulse response functions with unrestricted identification and identification horizon of 48 months



Note: See figure 8. The horizon used for identification of the news shock is now 48 months.